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DETERMINING RUN-OFF FROM RAINFALL

by

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For written discussion

SYNOPSIS

Various methods of determining the relation between effective rainfall and storm run-off are examined and shown to be particular cases of the general unit-hydrograph theory. A systematic approach to the investigation of the relation between the characteristics of a catchment and its response to rainfall is indicated.

Introduction

THE determination of the magnitudes and frequencies of discharges in sewers and in natural catchments drained by open streams by consideration of the amounts and frequencies of rainfall over the area has been the subject of many Papers published in the past half-century. It was perhaps inevitable that some confusion would arise, particularly since almost all such Papers deal exclusively with either urban or natural catchments. This has resulted in the growth of two different approaches to what is virtually the same problem. Failure to appreciate the identity of the problem (on both urban and natural catchments) has resulted in the public health engineer being deprived of the tools developed by the hydrologist, who at present is in a much more advanced position than his urban colleague.

2. It is the purpose of this Paper to review briefly the various methods developed over the years, to compare them with one another and isolate their common elements. As a result, what is believed to be a systematic method of investigation, suitable for either urban or natural catchments, emerges. This method is currently being applied by the Author to a series of natural catchments. In order, however, to distinguish more clearly between the method and its application in a particular case, as well as to avoid undue length, it is proposed to keep the results of the application for a later Paper.

Notation

A (with appropriate suffix) denotes area
a denotes an area within the catchment
C ,, coefficient of run-off
F ,, frequency

i denotes effective rainfall intensity
L (with appropriate suffix) denotes length
m is an empirical parameter
p denotes mean intensity of rainfall
Q (with appropriate suffix) denotes discharge
R ,, ,, ,, ,, quantity of rainfall
S ,, ,, ,, ,, storage
T ,, ,, ,, ,, period
T_c denotes time of concentration
t ,, time
U ,, unit hydrograph
τ ,, time ordinate
ϕ ,, storm frequency
ϕ' ,, first derivative of ϕ
ψ (with appropriate suffix) denotes frequency

THE PROBLEM

3. The relation between storm run-off and rainfall may be considered in three parts:—

(*a*) The relation between the volume of rainfall in a given storm, and the volume of storm run-off resulting.

(*b*) The manner in which the storm run-off is distributed in time. If $i = i(t)$ represents the distribution in time of effective rainfall intensity (i.e. rainfall minus all losses) and $Q = Q(t)$ represents the flow of storm run-off past the gauging station, then the transformation which the catchment performs on $i(t)$ to produce $Q(t)$ is the effect which must be found.

(*c*) The relation between rainfall frequency and discharge frequency.

If (*a*) and (*b*) are known, the hydrograph of storm run-off due to any given rainfall storm in any given circumstances can then be predicted. A means of predicting the frequencies of peak discharges from standard rainfall quantity-duration-frequency curves would still, however, have to be found.

4. The first part of the problem is of much less significance on urban than on natural catchments because the percentage run-off on the former, at least in intense storms, is sufficiently near 100 (and comparatively independent of antecedent conditions) to make errors from incorrect assessment of this factor comparatively slight. It is perhaps on this account that investigators working on urban catchments have generally been content to use a fixed percentage run-off, while hydrologists working on natural catchments have had to adopt more elaborate methods, e.g. the work of Linsley and Ackermann[1,2] in obtaining, by statistical analysis of the results of a large number of storms, correlations between the volume of run-off, and the volume of rainfall and indices representing the hydrological condition of the catchment at the time of occurrence of the storm.

5. The second part of the problem—the determination of the operation performed by the catchment on the input $i(t)$ to produce the response $Q(t)$—is

[1] The references are given on page 183.

almost the same problem on either urban or natural catchments. The relation is, in fact, more easily determined on urban catchments, because of the almost complete absence of ground-water flow or base flow, which, on natural catchments, must be separated from the hydrograph of total discharge, before the storm run-off due to any particular storm can be isolated. Furthermore, on natural catchments, the determination of the distribution in time of rainfall losses during a storm is rarely accurately possible. On urban catchments where the losses are small if not entirely negligible this difficulty scarcely arises. Despite these disadvantages the present position is that almost all progress to date has been made by hydrologists working on natural catchments, while public health engineers are still compelled to use (with minor modifications) the method introduced to Great Britain by Lloyd-Davies[3] in 1906.

6. The third part of the problem—the frequency relation—requires the prior solution of (*a*) and (*b*) (see § 3), and is thereafter purely analytical. Consequently its solution must be sought in the same way for both natural and urban catchments.

7. Parts (*b*) and (*c*) only will be dealt with in this Paper, assuming where necessary that the volumetric relation between rainfall and run-off can be achieved by multiplying the total rainfall by a run-off coefficient to obtain the effective rainfall. It will also be assumed that this coefficient can be predicted with sufficient accuracy from knowledge of local conditions. In fact, on any one catchment, this coefficient is not as variable as might be expected, particularly during rather large storms which effectively saturate the catchment during their earliest portions. This Paper will therefore be confined to discussing the following two problems:—

(*a*) Given $i(t)$, the intensity of effective rainfall as a function of time, to find $Q(t)$ the corresponding hydrograph of storm run-off, particularly the peak discharge.

(*b*) Given a rainfall frequency formula expressing frequency of given quantities of rain in given storm-periods, to find the frequency of a given peak discharge on a catchment for which a method is known of determining run-off from rainfall.

Historical development

The "Rational method"

8. The origin of this method is somewhat obscure. In Great Britain it is often referred to as the Lloyd-Davies[3] method and hence by implication ascribed to his Paper of 1906. It has been shown, however, by Dooge[4] that the principles of the method were explicit in the work of Mulvaney[5] in 1851. As currently understood[6] the method may be stated as follows. For every catchment there is a period, known as the time of concentration T_c, which is the time required for a particle of water to flow from the farthest part of the catchment to the gauging station. The discharge peak occurs when the whole catchment is contributing at the gauging station, i.e. a period T_c after start of rain, and is equal to the mean intensity of the effective rain during this period. This can be stated as:

$$Q = CAp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is the coefficient of run-off, A denotes the area, and p the mean intensity of rainfall during the period T_c. This formula is known in the literature as the "rational formula". Kuichling[7] in 1889 suggested that C approached a constant value for a given catchment as the magnitude of the storm increased. Kuichling's purpose was not so much the prediction of the hydrograph of run-off from the rainfall but rather the determination of the frequency of discharges from the frequency of rainfall. A set of curves may be assumed, giving the frequency of any given quantity of rainfall in any given time or less as $F=F(R, T)$ (i.e. frequency as a function of quantity and period in which the quantity is to be expected). Such curves are the standard rainfall quantity-duration-frequency curves. Given this information and assuming C and T_c to be known for a given catchment, then the frequency of any given discharge can be obtained by reading from the rainfall frequency curves, the frequency of the quantity of rain required in T_c to produce Q.

9. In detail, to produce a peak discharge Q the effective rainfall must have a mean intensity equal to Q over the period of concentration T_c, i.e. a volume of effective rainfall T_cQ $\left(\text{corresponding to a volume of actual rainfall } \frac{T_cQ}{C}\right)$ must occur in time T_c. The rainfall frequency chart or formula gives directly the frequency of this quantity in the given time T_c. This frequency is the frequency of Q, because for every occurrence of Q there will also, according to the rational theory, be an occurrence of $\frac{T_cQ}{C}$ in. of rainfall in period T_c, and conversely. It is important to appreciate that the reason why the frequency of Q is equal to the frequency of a certain amount of rainfall in a certain time is that, according to the rational theory, the discharge Q depends on the quantity of rainfall in a certain critical period. If on the other hand it was assumed that a quantity R_1 in a period T_1, or a quantity R_2 in a period T_2, or a quantity R_3 in a period T_3 could each produce Q, then it would be necessary to add the frequencies of the independent occurrences of these quantities (R_1 in T_1, R_2 in T_2, etc.) to obtain the frequency of Q. It is also worthy of note that if C be assumed to vary with the conditions in the catchment at the time of occurrence of the storm, then the rational method of determining the frequency would not work, since the peak discharge Q would no longer be related to a unique quantity of rainfall in a unique time, but could be produced by R_1 in T_c or R_2 in T_c depending upon the value of the coefficient of run-off occurring at the time of the storm.

10. While it is adequate to assume that the peak discharge is directly proportional to the total volume of rain falling in the time of concentration—adequate, that is, in the sense that no further assumptions are required in the theory—it can be shown that this assumption is equivalent to a number of other assumptions which are known to be inaccurate. This will be dealt with later when a unified view is taken of the various methods proposed since the advent of the rational theory.

The tangent method

11. An inconsistency was discovered in the rational method in that it sometimes gave a greater discharge, for a given frequency, for part of a catchment than for the whole. For the catchment shown in Fig. 1 the peak discharge can be calculated for any given frequency by the rational method:— If the area and time of concentration of the catchment from the outfall A to B be A_1 and T_1 respectively, and A_2 and T_2 represent the values of these quantities for the whole catchment from A to C, then by the rational method, the discharge from the

catchment between A and B is given by $Q_1 = Cp_1A_1$ where p_1 denotes the mean intensity of rainfall to be expected with the given frequency lasting over a period T_1. Similarly $Q_2 = Cp_2A_2$ can be found for the whole area. Now since T_2 is much greater than T_1, p_2 is much less than p_1, and it is possible that the ratio of p_1/p_2 is greater than the ratio A_2/A_1, in which case Q_1 is greater than Q_2. This is an inconsistency which derives from the inexactness of the fundamental assumption, namely that Q is equal to the mean intensity of effective rainfall during the time of concentration.

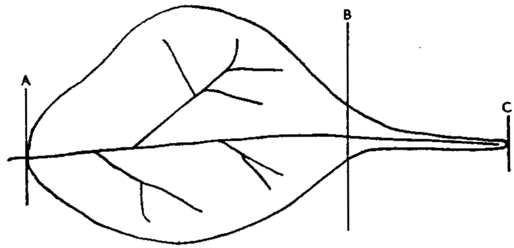

FIG. 1

12. Reid[8] in 1927 and Norris[9] in 1946 put forward graphical methods whereby that part of any given catchment could be recognized which gave the greatest discharge by the rational method, using the Ministry of Health rainfall curve to obtain the intensity appropriate to any given time of concentration.

13. These corrections of course provided no fundamental change in the rational method, and while to some extent correcting the effect, they in no way removed the cause of the error. The methods of Reid and Norris, which are graphical, are well known and do not require a detailed description here. The word "tangent" in the name of the method derives from the fact that a tangent is drawn in the course of the graphical calculation.

The time-area methods

14. A large number of these methods have been presented from time to time, many of then differing only in the method of presentation. At this stage it is well to recall the two distinct problems enumerated in § 7: (*a*) the determination of the hydrograph, and particularly the peak discharge, due to a given effective rainfall-time distribution, and (*b*) the determination from rainfall frequency curves of the discharge having a given frequency. The time-area methods are all identical when applied to the determination of the run-off from a given effective rainfall distribution. Their specific differences lie in their assumptions concerning the variation of intensity of effective rainfall within the duration of the storm assumed to be the storm having the frequency required.

15. The central idea of all the time-area methods is of a time contour, or line of equal time of flow to the gauging site progressively enclosing more and more of the catchment area as the time of flow to the outfall is increased, until eventually, at the "time of concentration" the whole catchment is contributing. A plot of area enclosed by a time contour against time is known as "the time-area concentration curve". Ross[10] in 1921, was perhaps the first to suggest this idea.

16. The application of the method is most easily explained by an example. The run-off at any time t is equal to the area enclosed by the 1-hour contour multiplied by the mean intensity of effective rainfall over the hour previous to t, plus the additional area enclosed by the 2-hour contour multiplied by the mean intensity of effective rainfall during the period from 1 to 2 hours before t and so on. In general the discharge at any time t is given by:

$$Qt = \sum_{r=1}^{r=n} i(n-r)\,a_r \qquad \ldots\ldots\ldots \quad (2)$$

assuming the time of concentration to be divided into n intervals. The mean intensity of effective rainfall is $i(n-r)$, in the interval $n-r$, and a_r is the area enclosed by the contours bounding the rth interval.

17. In order to apply this idea to calculating the discharge corresponding to any given frequency, Ross suggested that a storm should be assumed of which the first period t contained the quantity of rainfall given by the appropriate rainfall quantity-duration frequency curve in time t. For example, the first ½-hour would contain the quantity of rainfall appropriate to a ½-hour and a frequency of say one per year, if it was wanted to calculate the once-per-year peak discharge. Similarly the first hour would contain the amount appropriate to 1 hour and so on. Hawken,[11] when discussing Ross's Paper, suggested that the diminishing intensity of rainfall chosen by Ross might not give the greatest discharge and he proposed that the (say ½-hour) periods of uniform intensity should be rearranged so as to give the greatest discharge. This is also the idea suggested by Judson.[12]

18. Ormsby[13] very rightly points out that to associate a frequency equal to the rainfall quantity-duration curve to the discharge peak Q calculated by Judson's method would contain an error of overestimation caused by the arbitrary arranging of the intensities. In fact, even Ross's system overestimates, because while the rainfall frequency curve for the required frequency gives a different rainfall intensity for each different duration, it is entirely incorrect to think of these intensities occurring together.

19. This can best be understood by considering a specific example:—The rainfall frequency curve may give the information that once per year an intensity of ¼ in/hour for 6 hours, also ½ in/hour for 2 hours, and 1 in/hour for 1 hour, etc., may be expected. This does not mean that these three occurrences will occur in the same storm; they may, or may not. To assume that they do, overestimates the magnitude of Q corresponding to any given frequency. To assume that they occur together, and that, furthermore, the arrangement of them among themselves is such as will give the greatest discharge, removes any remaining connexion between the frequency of the discharge so calculated and the nominal frequency of the rainfall curve. This was pointed out by Gregory and Arnold[14] in 1932. Ormsby removes some of the overestimate by using an arbitrarily chosen storm pattern, i.e. the arrangement of the various intensities throughout the storm is not chosen so as to obtain the maximum value of the run-off, but the storm does contain, in any period t, the quantity of rainfall which the Ministry of Health formula gives as occurring in that period. Consequently Ormsby's method also overestimates.

20. It is seen, then, that all the time-area methods are identical if used to calculate the discharge from any given catchment for any given distribution in time of effective rainfall. They differ in the discharge they give as having a

frequency equal to that of a given rainfall-intensity-duration curve. The differences exist because, unlike the rational method, which gives the same discharge irrespective of the distribution of the rainfall within the time of concentration, these methods give different discharges depending on the variation of the intensity of the effective rainfall and consequently require arbitrary decisions concerning this variation.

The unit-hydrograph theory

21. In 1932 Sherman[15] formulated the unit-hydrograph theory which, with some modification, is now the almost universally accepted method of determining the run-off hydrograph from the effective rainfall. He suggested that the run-off hydrograph due to 1 in. of effective rainfall generated uniformly in space and time over a catchment, in unit period, was characteristic of the catchment. This curve he called the unit hydrograph. He postulated: (i) that the hydrograph from n units of effective rainfall generated uniformly over the catchment in the same unit period could be obtained by multiplying the ordinates of the unit hydrograph by n; and (ii) that the run-off due to two or more such periods of effective rainfall could be obtained by adding the hydrographs obtained from each.

22. The unit hydrograph is then the hydrograph due to a "block" of effective rainfall of unit volume and duration, and the hydrograph due to any effective rainfall distribution can be built up by replacing, as it were, each unit block of effective rainfall by the unit hydrograph. Ordinarily the storm run-off is expressed in cusecs per 1 sq. mile and the unit volume of effective rainfall is usually taken as 1 in. The unit duration of the storm is often varied and reference is made to a 1-hour unit hydrograph, a 1-day unit hydrograph, etc., being the hydrographs of storm run-off due to 1 in. of effective rainfall generated uniformly in space and time over the catchment, in 1 hour, 1 day, etc. The instantaneous unit hydrograph will frequently be referred to. It is the limit to which the short-period unit hydrograph tends as the period of the effective rainfall is diminished indefinitely.

The modern form of the unit-hydrograph theory[16–19]

23. This can be simply stated in one sentence: "It is assumed that the storm run-off is derived from the effective rainfall by a'linear operation'". The conception of a linear operation is common in other branches of engineering, particularly in servo-mechanisms and the anaylsis of electrical networks. In terms of rainfall and run-off, the idea is that the run-off hydrograph due to several equal periods of effective rainfall is obtained by adding simultaneously occurring ordinates of the run-off due to each period of effective rainfall, and the ordinates of run-off due to any one period of effective rainfall are proportional to the volume of effective rainfall in that period. This will be seen to be the simplest possible assumption of the relation between effective rainfall and storm run-off. In Fig. 2 the first period of effective rainfall produces a run-off hydrograph as shown, the second period would, on its own, produce a similar hydrograph differing only in that corresponding ordinates would be later by the difference in time between the two periods of effective rainfall, and in having ordinates proportional to the volume of effective rainfall in the second period.

24. The assumption of linearity implies that the run-off hydrograph due to the two rainfall periods is simply the sum of those due to each period. In operational mathematics the response of a system to one very short period of input of unit volume is known as the indicial response of the system to unit impulse.

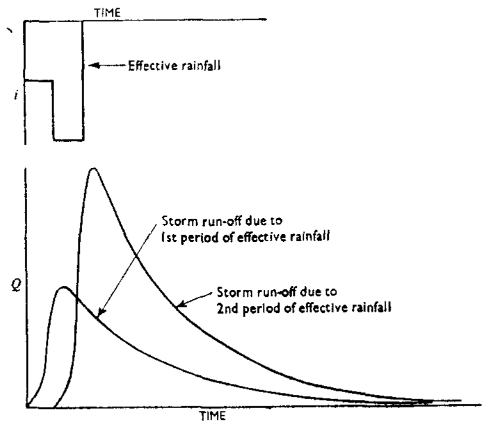

FIG. 2

It is more customary to take, as the indicial response, the response of the system to "unit step input", i.e. a step of unit magnitude in the input, in this case a step of 1 cusec in the intensity of effective rainfall. Either of these "indicial responses", known in hydrological literature as the instantaneous unit hydrograph and the *S*-curve respectively, may be taken as indicating the operation which the catchment performs on the effective rainfall to produce the storm

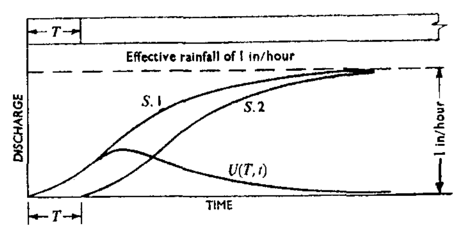

FIG. 3

run-off. The relation between the two responses can very easily be established. In Fig. 3 the uniform effective rainfall of intensity unity produces the *S*-curve marked S_1. S_2 is the same curve moved to the right by T hours, and may be looked upon as being due to uniform effective rainfall of intensity unity beginning T hours later than that which produced S_1.

25. By the principle of linearity the vertical ordinates between the two *S*-curves give the ordinates of run-off due to the effective rainfall for the period *T*, i.e. for a volume of effective rainfall equal to *T* hours by 1 in/hour, i.e. *T* in. Consequently the unit hydrograph of period *T* is obtained by dividing the difference between the two *S*-curves by *T* in., i.e.:

$$U(T, t) = \frac{1}{T}(S_t - S_{t-T}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now as *T* is diminished indefinitely $U(T, t)$ approaches the instantaneous unit hydrograph and the right-hand side of the equation approaches the derivative of the *S*-curve:

$$U(0, t) = \frac{d}{dt}.S_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

i.e. the ordinate at time *t* of the instantaneous unit hydrograph is equal to the derivative of the *S*-curve with respect to time at time *t*.

26. It will now be seen how either of these indicial responses, when known, can be used to predict the run-off due to any given effective rainfall. Fig. 4 gives: (*a*) a plot of the variation with time of the intensity of the effective rainfall; (*b*) the instantaneous unit hydrograph; and (*c*) the hydrograph of storm run-off.

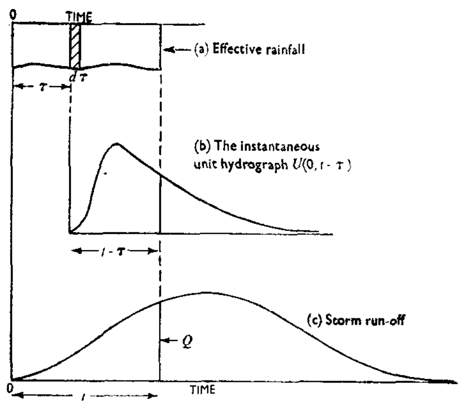

FIG. 4

The effective rainfall during a short period $d\tau$ at time τ will produce a run-off at time *t* equal to the volume of effective rainfall in the period $d\tau$ multiplied by the ordinate of the instantaneous unit hydrograph at time $t-\tau$,

i.e. $$dQ_t = U(0, t-\tau)\, i(\tau) d\tau$$

$$Q_t = \int_0^t U(0, t-\tau)\, i(\tau) d\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation, known in mathematical literature as Duhamel's integral, is taken as the definition of linearity. It is also of recent years becoming accepted as the definition of the unit-hydrograph theory.[16–19] The rational method and the time-area method will now be re-examined in the light of the unit-hydrograph theory.

The rational method

27. It is assumed that the time of concentration is constant for a given catchment. It is also assumed that the run-off peak is directly proportional to the mean intensity of effective rainfall in the time of concentration. Consequently linearity is assumed. This method is therefore a particular case of the unit-hydrograph method. If equation (1) is written as an integral, the relation between it and equation (5) becomes obvious. Equation (1) becomes:

$$Q = A\frac{\int_{t-T_c}^{t} i.d\tau}{T_c} = \int_{t-T_c}^{t} \frac{A}{T_c} i(\tau)d\tau$$

Comparing with equation (5), it is seen that if $U(O, t-\tau)=\frac{A}{T_c}$ within the range $\tau=0$ to $\tau=T_c$, and is zero outside this range, the two systems are identical. But $U(O, t-\tau)$ is the ordinate of the instantaneous unit hydrograph at time $t-\tau$. Therefore the rational method is identical with the unit-hydrograph method provided an instantaneous unit hydrograph of constant ordinate over a period T_c (i.e. a rectangular instantaneous unit hydrograph, Fig. 5) is assumed.

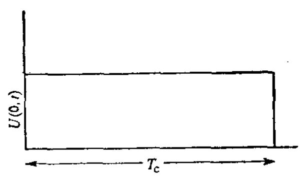

FIG. 5

28. This analysis brings to light the weakness of the rational theory. It is inconceivable that the response of a catchment to an instantaneous rainfall could be a continuous run-off of constant value during the period T_c beginning and ceasing abruptly. This, however, is the inescapable assumption upon which the theory rests.

The tangent method

29. These modifications do not change the basic assumption of a rectangular instantaneous unit hydrograph. As already shown, they merely produce a graphical means of finding the part of the catchment which by the rational method and the Ministry of Health rainfall curve will give the greatest discharge.

The time-area method

30. It has already been shown that this method assumes that the rate of run-off from any elementary area within the catchment is equal to the intensity of the effective rainfall on that area, and contributes to the outflow at the gauging station at a time later than the effective rainfall by the time of concentration of the elementary area. Since the time of concentration of each elementary area is assumed independent of the rate of run-off from the elementary area and from surrounding areas, it is again a linear system. As with the rational method, comparison of the corresponding equations will show what assumption is made as to the form of the instantaneous unit hydrograph.

31. The effective rainfall at time τ generated on an area whose time of concentration is $t-\tau$ (see Fig. 6) will contribute to the outflow at the gauging station at time t.

$$dQ = i(\tau) \times da_{(t-\tau)}$$

$$dQ = \frac{da_{(t-\tau)}}{d\tau} i(\tau) d\tau$$

$$Q = \int_0^t \frac{da_{(t-\tau)}}{d\tau} i(\tau) d\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Comparison of equation (6) with equation (5) shows that the instantaneous-unit-hydrograph term in the latter is replaced by the term $\frac{da}{d\tau}$ in the former. Consequently the time-area methods all assume that the derivative of the time-area concentration curve with respect to time furnishes the instantaneous unit hydrograph.

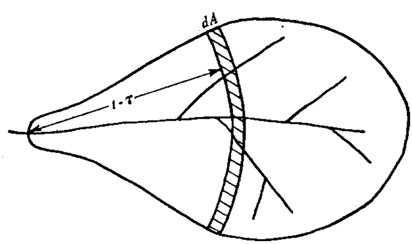

FIG. 6

32. The inaccuracies involved in this assumption and in the corresponding assumption of the rational theory can be seen by reference to Fig. 7, which shows three instantaneous unit hydrographs empirically derived by O'Kelly.[20] The rivers of these three catchments are sufficiently canalized to permit the calculation of the time of concentration and the time-area concentration curve by the Manning formula. The discharge at each section was taken as the "bank full" discharge thus enabling the velocity and time of flow to be estimated for each reach of river. Fig. 7 shows the three empirically derived unit hydrographs

compared with the instantaneous unit hydrographs according to the rational and the time-area methods. The great difference between the observed and assumed instantaneous unit hydrographs emphasizes the inaccuracy of the rational method and the time-area method at least when applied to natural catchments.

The evidence for the unit-hydrograph theory

33. Having examined the various methods, the discovery of their common factor—the assumption of linearity—prompts an inquiry into what evidence is available to justify this basic assumption. Surprisingly enough, there is comparatively little direct evidence. The reason for this is that while it is a comparatively routine matter to derive a unit hydrograph for any chosen period for a catchment, if the distribution in time of effective rainfall and the resulting hydrograph of storm run-off are given, it is by no means a simple matter in the case of natural catchments to determine either the part of the actual rainfall-time curve which is effective or the part of the hydrograph which is caused by any one storm. The matter is, however, by no means hopeless; in fact, unit hydrographs can generally be derived, subject to some degree of error, for any catchment for which a record of rainfall and stream flow is available.

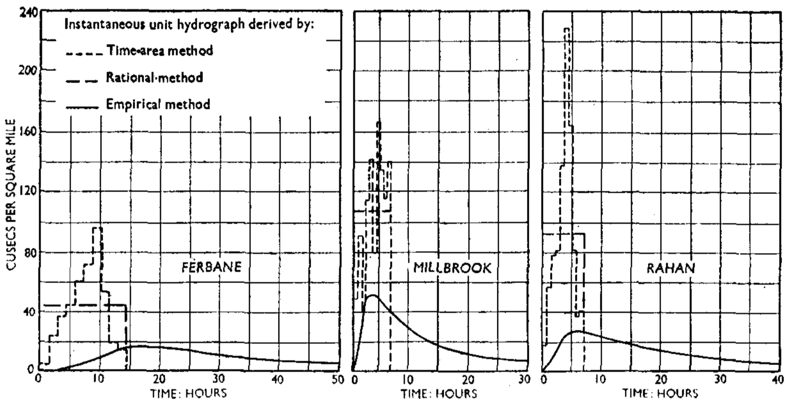

FIG. 7

34. Frequently unit hydrographs can be derived for catchments where no rainfall record is available, by choosing floods which the sudden rise in the hydrograph indicates were caused by short intense storms. Unit hydrographs derived from such floods may safely be taken as approaching closely the instantaneous unit hydrograph. Generally the various unit hydrographs derived from any one catchment compare fairly satisfactorily with one another. It appears certain, therefore, that the inaccuracies involved are much more likely to be attributable to the uncertainties of the derivation than to a failure of the theory.

The real justification for the unit-hydrograph theory is that the simplifying assumption of linearity cannot be replaced by a non-linear relation (at least for natural catchments), until evidence in the form of much more accurate data than at present exist becomes available and points to a more complex relation.

35. It is sometimes argued that the unit-hydrograph theory is in contradiction to the fundamental laws of hydraulics, because it depends on the assumption that the velocity of flow at any place is independent of the depth of flow. This objection is, no less, a valid criticism of any of the other linear systems. Without wishing to engage in controversy on this matter, since it is freely conceded that linearity is assumed without proof, the Author suggests that the objection springs from an oversimplified view of the method by which the flood reaches the gauging station. The routing of a flood through surface storage and channel storage is of more concern than the movement of particles of water. If the catchment is viewed as a network of such elements of storage, some of reservoir type and some of channel type, one discharging into the next and branching in any conceivable way, and if it is assumed in accordance with current flood routing practice (e.g. the Muskingum method) that the storage in each of these elements is either a linear function of the discharge from it (reservoir type) or a linear function of a weighted mean of inflow and discharge (channel type), then the operation of the whole catchment is linear, and the unit-hydrograph theory is exact. While it is agreed that the assumption of linearity in the storage discharge equations is, in general, an over-simplification, it is not necessarily contrary to the laws of hydraulics.

THE RELATION BETWEEN THE CHARACTERISTICS OF A CATCHMENT AND ITS INDICIAL RESPONSE

36. In order to determine the storm-run-off hydrograph resulting from a given distribution in time of effective rainfall on a catchment whose characteristics are known, it is necessary to establish the relation between the indicial response and the catchment characteristics. The most straightforward way of establishing this relation is to derive unit hydrographs for any chosen period (instantaneous unit hydrographs, 12-hour unit hydrographs, *S*-curves, etc.) for each of several catchments, and to plot measured characteristics of the catchments and the unit hydrographs against one another in the hope of finding one or more well established relations. These relations may then be used to determine the unit hydrograph for the chosen period for any catchment, from knowledge of the catchment characteristics alone.

37. To do this it is obviously necessary to describe the indicial response of the catchment in numbers, e.g. the value of the peak of the instantaneous unit hydrograph, the lag from the centre of area of the effective rainfall to the centre of area of the storm run-off, etc. It is desirable that the parameter or parameters chosen should be, according to the unit-hydrograph theory, constant for any one catchment. These two suggested parameters satisfy this requirement. Other parameters sometimes used do not do so, e.g. the lag from the centre of area of the effective rainfall to the peak of the storm run-off (see reference 21). According to the unit-hydrograph theory this quantity depends on the duration

of the effective rainfall. If a single parameter is chosen it must be assumed, in order to complete the indicial response, that the form of the indicial response is a function solely of the chosen parameter, e.g. Commons[22] suggested that the "basic hydrographs" derived from all short-period unit hydrographs were identical. The basic hydrograph may be defined as the curve obtained by dividing all the ordinates of the unit hydrograph by the value of the peak ordinate and multiplying all abscissae by the peak divided by the volume of the unit hydrograph. In a sense the basic hydrograph is a representation of the shape of a unit hydrograph. Commons suggested that, as a first approximation, all short-period unit hydrographs were of the same basic shape and differed only in scale; some being high and of short duration, others being low and of long duration. If Commons's assumption is accepted, then knowledge of one parameter alone, say the peak of the instantaneous unit hydrograph, enables the instantaneous unit hydrograph, and consequently the unit hydrograph of any period, to be defined completely. On the other hand, the basic shape itself may be found to vary with the value of the chosen parameter. In such a case the indicial response could still be described completely by the value of the single parameter, by using along with it the appropriate basic shape.

38. If it was found that the basic shape varied with a different measure of the catchment characteristics from that which determined the chosen parameter, then the values of at least two different parameters, each of which must be correlated separately with the catchment characteristics, would be required to complete the indicial response. Some of the best known correlations will now be examined, principally to see what parameters have been used to express the indicial response and also to see what characteristics of the catchments have been found significant.

39. It has already been seen that the rational method is equivalent to the assumption that the instantaneous unit hydrograph is a rectangle of base equal to the time of concentration of the catchment. The time-area methods assume that the ordinates of the instantaneous unit hydrographs are proportional at corresponding times to the first derivative of the time-area concentration curve. These connexions between the catchment and its indicial response are, however, little more than assumptions and are not the results of empirically derived correlations. Fig. 7 shows that on natural catchments at least, these assumptions are far from accurate.

Bernard's approach[23]

40. The first recorded attempt at a correlation was made by Bernard. His basic data consisted of daily rainfall records and the daily discharges from six catchments of areas ranging from 500 to 6,000 sq. miles. Bernard assumed that the peak of the unit hydrograph should be inversely proportional to the time of concentration. This period might be assumed to be proportional to the length of the longest channel divided by the square root of the slope. Bernard, in fact, followed Gregory and Arnold,[14] and used a modified formula for the time of concentration. With this formula he calculated, for each catchment, a factor *U*,

which was assumed to be proportional to the time of concentration, and he plotted U against the ordinates of the 1-day unit hydrograph at 1 day, 2 days, etc. after the rainfall. These plottings were made on logarithmic paper and the points approximated to by parallel straight lines. One straight line was drawn to approximate to all the points representing ordinates of the 1-day unit hydrograph on the first day after the rainfall; another line was drawn for points representing the ordinates on the second day, etc.

41. To obtain the 1-day unit hydrograph for a catchment the value of U for that catchment is calculated and entered on the chart. Then the ordinates on the first day, the second day, etc., after the rainfall, can be read off using the appropriate lines on the chart. Clearly, since only one measure of the catchment was used, this measure must determine both the scale and basic shape of the unit hydrograph. Consequently it is implicitly assumed in Bernard's correlation that all 1-day unit hydrographs, having the same peak, are identical.

McCarthy's approach

42. This work has not been published by McCarthy, but an account of it is available.[21] The basic data consisted of twenty-two 6-hour unit hydrographs for catchments of areas ranging from 74 to 716 sq. miles. The catchment characteristics used were area, overland slope, and stream pattern. Area was accounted for by converting the unit hydrographs and the catchment characteristics to the corresponding quantities for catchments of 10 sq. miles by using the Froude model law. In order to calculate the overland slope, the area above each contour was plotted against the level of that contour and the mean slope of this area-elevation curve was taken as the overland slope. This quantity, being area divided by length, had the dimension of length and had, therefore, to be multiplied by the length scale in converting to the model catchment.

43. The stream-pattern number was defined as having a value of 1 if no stream had a tributary draining more than 25% of the total catchment area; a value of 2 if there were two tributaries of approximately equal size draining at least 50% of the total catchment area; and a value of 3 if three tributaries drained 75% of the total catchment area. The peaks of the unit hydrographs were plotted on logarithmic paper against the slopes of the respective catchments and each point was marked with the appropriate stream-pattern number. For each of the three stream-pattern numbers, a curve was drawn, to fit as well as possible the points of that number. The peak of the unit hydrograph for any catchment could be obtained by entering the chart with the value of the slope and reading from the curve of appropriate stream-pattern number. McCarthy did not attempt to correlate any other characteristic of the unit hydrograph with the catchment characteristics; instead he correlated the lag from the beginning of rainfall to the peak, and the time base of the unit hydrograph separately, with the unit-hydrograph peak. In expressing the lag and time base of the unit hydrograph as functions of the peak, he, like Bernard implicitly assumed that all unit hydrographs having the same peak are identical.

Snyder's approach[24]

44. Snyder neglected catchment slope and correlated the lag from the centre of area of the effective rainfall diagram to the peak of the storm run-off against the product LL_{cA}, where L denotes the length of the longest watercourse to the catchment boundary and L_{cA} denotes the length, by the channels, to the centre of area of the catchment. Like McCarthy, Snyder found correlations between the lag and other measures of the unit hydrograph. Consequently he also implied that all unit hydrographs having the same lag were identical.

Taylor and Schwarz approach[25]

45. The basic data used by Taylor and Schwarz consisted of twenty catchments of areas ranging from 20 to 1,600 sq. miles and, for each catchment, several unit hydrographs of different periods. The catchment characteristics used were those used by Snyder, with the addition of the average slope of the main channel. The peaks, of the unit hydrographs of different periods, for each catchment were correlated with the periods of the unit hydrographs and a set of curves obtained of form $U(T, P) = U(O, P)\, e^{mT}$ where $U(T, P)$ is the peak of the unit hydrograph of period T; $U(O, P)$ is the peak of the instantaneous unit hydrograph; e is the base of the natural logarithms and m is an empirical parameter, constant for each catchment. Since the relation between the peak of the unit hydrograph of period T and the peak of the instantaneous unit hydrograph is a function of the shape of the instantaneous unit hydrograph, the value of m is clearly an index of the shape of the instantaneous unit hydrograph.

46. The next step in the correlation was to plot $U(O, P)$ and m separately, against the catchment characteristics. The equations obtained were $U(O, P) \propto 1/\sqrt{S}$ and $m \propto (LL_{cA})^{0\cdot3}$, i.e. the peak of the instantaneous unit hydrograph was found to be a function of the main channel slope, and the shape of the instantaneous unit hydrograph was found to be a function of the catchment length.

O'Kelly's approach[20]

47. O'Kelly assumed that the instantaneous unit hydrograph could be obtained by routing an isosceles triangular inflow, of the correct volume and of base length T hours, through storage described by $S = KQ$. Clearly the instantaneous unit hydrograph obtained depended on both T and K. The catchment characteristics used were area and overland slope. The effect of area on T and K was assumed to be given by the Froude model law and all values of T and K were modified to correspond with a catchment of 100 sq. miles area. The modified values of T and K were plotted against the overland slope which was defined as the median value of the maximum slope occurring at the intersections of a grid of square mesh imposed on a map of the catchment.

48. O'Kelly's conclusion was that the modified T and K could both be expressed as monomial powers of the slope, i.e. $T = AS^B$ and $K = CS^D$ where S denotes the slope and A, B, C, and D are empirically derived constants. If B and

D were equal then T/K would be a constant A/C, and the "shape" of the instantaneous unit hydrograph would have been fixed as suggested by Commons. In fact O'Kelly used slightly different values of B and D and so obtained a basic shape which varied slightly with the catchment slope, and consequently with the unit-hydrograph parameter K.

49. The Author had the privilege of working with the late J. J. O'Kelly and later had an opportunity of re-working over the basic data with J. P. Farrell of the Office of Public Works, Dublin. The conclusion then reached was that the evidence for varying the basic shape with catchment slope was inadequate, and practically equally good results could be obtained by using Commons's basic shape and varying a single parameter with the catchment slope. In either case, since T and K were both correlated with the single catchment characteristic slope, it was implicitly assumed that T was a function of K and consequently the indicial response of the catchment could be described by the value of a single parameter.

Clark's approach[26]

50. A somewhat different approach to the determination of the relation between the instantaneous unit hydrograph and the catchment was suggested by Clark. He reasoned that the ordinates of the instantaneous unit hydrograph should be proportional to those of the derivative of the time-area concentration curve at corresponding times if no storage existed in the catchment. He therefore routed an inflow of the same form as the derivative of the time-area concentration curve through a single reservoir which was assumed to represent the damping effect of the storage distributed throughout the catchment. The method requires knowledge of two quantities, T_c and K the constant in the storage equation $S = KQ$. Clark suggested that the parameters might be related. However, the main interest of the method is that if T_c and K were correlated separately with the catchment characteristics the full instantaneous unit hydrograph could be obtained from T_c and K and the time-area diagram for the catchment. When an inflow whose ordinates are proportional to those of the derivative of the time-area concentration curve at corresponding times is routed through linear storage, where T_c and K are of the same order, the outflow obtained has the same general shape as a typical unit hydrograph with the exception that the outflow is usually somewhat less smooth.

51. In order to test Clark's theory properly it is necessary to find out if the irregularities observed in the outflow in fact occur in the actual instantaneous unit hydrograph. However, due to imperfections in the data, it is never really possible to do this, because in deriving short-period unit hydrographs, particularly the instantaneous unit hydrograph, it is almost always necessary to smooth the derived curve to prevent "hunting". If, on the other hand, long-period unit hydrographs derived from the records of floods are compared with the unit hydrograph for the same period derived from Clark's instantaneous unit hydrograph, it will be found that many of the irregularities of Clark's instantaneous unit hydrographs are smoothed out and do not appear, or appear very much diminished, in the long-period unit hydrographs. This leads to the conclusion

that, like the unit-hydrograph theory itself, Clark's theory can never be adequately tested on natural catchments.

52. Unless and until such test has been made it seems somewhat arbitrary and needlessly complicated to proceed along Clark's lines. The mere finding of correlations between T_c and K and the catchment characteristics which would permit the reasonably close reproduction of actual short-period unit hydrographs by Clark's method would not be sufficient evidence to justify the method, unless it was also observed that peculiarities in the shape of the actual unit hydrographs were also reproduced.

53. O'Kelly's method which, in fact, was derived from Clark's, differs from it in replacing the derivative of the time-area diagram by an isosceles triangle of the same base and area. The results obtained were as good as those obtained by using Clark's method. Although O'Kelly obtained smooth instantaneous unit hydrographs, whereas Clark's were somewhat less smooth, the differences between the unit hydrograph of finite period derived by both methods were slight. Consequently O'Kelly's work showed that, owing to the damping effects both of the routing and of deriving a unit hydrograph of finite period from an instantaneous unit hydrograph, the irregularities in the shape of the derivative of the time-area concentration curve were so far diminished and smoothed out in the final product that one could obtain almost identical results by using an isosceles triangle instead of the derivative of the time-area concentration curve. This does not mean that O'Kelly disproved Clark's theory of how the instantaneous unit hydrograph is related to the catchment, but he showed that it would be very difficult indeed to test the theory on natural catchments, and, in the meantime, apparently equally good results could be obtained by relating a single parameter of the indicial response to the catchment characteristics and using a basic shape of instantaneous unit hydrograph which varied slightly with K.

54. The method used by Watkins[27] and described by him as a modification of Ormsby and Hart's method, wherein the run-off is calculated by the time-area method and then routed through storage, is, in fact, a "Clark" method. It is identical with taking the instantaneous unit hydrograph as being the outflow obtained by routing an inflow whose ordinates are proportional to those of the derivative of the time-area concentration curve through the hypothetical storage reservoir.

55. Looking back on the various methods which have been examined for relating the indicial response to the catchment characteristics, it is seen that Bernard, McCarthy, Snyder, O'Kelly, and Commons assumed that one parameter was sufficient to describe the indicial response of the catchment, and consequently all instantaneous unit hydrographs of the same peak were identical. Bernard found that this parameter was related to a measure of the catchment which was supposed to represent its time of concentration. McCarthy found that the indicial response was determined by the area, stream-pattern number, and overland slope of the catchment. Snyder found that the length of the catchment alone determined the indicial response and O'Kelly found that the area and slope of the catchment were the determining characteristics. Taylor and Schwartz alone of those mentioned, found it necessary to describe the indicial

response by two parameters. They found that the peak of the instantaneous unit hydrograph was determined by the slope of the main channel and its "basic shape" by the length of the catchment.

THE RELATION BETWEEN RAINFALL FREQUENCY AND DISCHARGE FREQUENCY

56. Assuming that for a certain catchment, a method is available by which the peak discharge due to any given rainfall in any given conditions can be determined, then Q (the peak discharge) is a known function of R, T, A, B, C, etc., where R and T denote quantity and duration of rainfall and A, B, C, etc. are measures of the other factors which enter into the relation. A might express the variability of intensity during the storm and B might be an index of the soil moisture deficiency at the time of the storm. C might represent the groundwater discharge. This relation may be expressed as $R=f(Q, T, A, B, C)$, that is, the quantity of rainfall required to produce a peak discharge Q is a function of Q, T, A, B, and C.

57. It is first assumed that there is available for the catchment a set of curves or an equation, expressing the frequency of any given quantity of rainfall in any given storm duration or less as $\phi=\phi[R, T]$. This is the standard rainfall-frequency formula. Details of the frequency distributions of A, B, and C are also required. It is then assumed that $\psi_1(A)dA$ is the frequency of a value of A between $A\pm\frac{dA}{2}$, and that $\psi_2(B)$ and $\psi_3(C)$ have corresponding meanings. The first step in developing a flood frequency formula is to convert the standard rainfall-frequency formula to a more useful form by differentiation (Farrell[28]). This gives $\phi'[R, T]dt$ as the frequency of storms of amount R or greater and of duration between $T\pm\frac{dt}{2}$. Since any one storm can provide several entries in the original rainfall-frequency data this method of deriving $\phi'[R, T]dt$ is not precisely the frequency required (Chow[29]) but it appears to be the best readily available method.

58. Two principles will now be applied: (*a*) that the frequency of Q is the sum of the frequencies of all possible combinations of R, T, C, and D, which produce Q, and (*b*) the frequency of the joint occurrence of any one set of values of R, T, C, and D is the product of their respective frequencies. This latter principle requires that the variables be mutually uncorrelated, which, in the absence of any evidence to the contrary, is the obvious assumption to make. If there is evidence of mutual correlation either positive or negative it must be allowed for. Applying these two principles a multiple integral for the frequency of Q is obtained:

$$F_Q=\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \phi'[f(Q, T, A, B, C), T]\psi_1(A)\psi_2(B)\psi_3(C)dA.dB.dC.dT$$

which can be integrated graphically for known values of f, ψ_1, ψ_2, and ψ_3. The

Author has applied this equation in a particularly simple case[30] with reasonable results.

Conclusions

59. An investigation into the relation between effective rainfall and storm run-off on either natural or urban catchments should isolate one parameter of the indicial response of each of several catchments. A correlation should be established between the values of this parameter and the characteristics of the catchments. The indicial response of each catchment should then be expressed in terms of the chosen parameter and if significant differences between the responses so expressed are observed, these should be correlated firstly, with the values of the chosen parameter and, if necessary, with other catchment characteristics.

60. The choice of indicial response and of the parameter to represent it is largely arbitrary, but the instantaneous unit hydrograph and the peak of the instantaneous unit hydrograph respectively would appear to be suitable. Consideration might also be given to using the time from the instant of effective rainfall to the centre of area of the instantaneous unit hydrograph as the parameter instead of the peak discharge. This quantity is very easy to measure accurately for, by the principle of linearity, it is also the time interval between the centre of area of the effective rainfall and the centre of area of the resulting storm run-off.[31] It might be called the "average delay time" of the catchment and one would certainly expect to find a correlation between it and the time of concentration as ordinarily measured. Assuming that this was done and a satisfactory correlation established, the next step would be to express the instantaneous unit hydrograph in terms of the average delay time by dividing all abscissae of the instantaneous unit hydrograph by the average delay time and multiplying all ordinates by this quantity divided by the volume of the instantaneous unit hydrograph. This would furnish a set of dimensionless unit hydrographs which might be expected (*cf* Commons), to a first approximation, to be identical.

61. A second-order refinement would be to find correlations between the values of parameters of the dimensionless unit hydrographs and, in the first instance, the average delay time, and if this was not adequate, the catchment characteristics themselves. A suitable parameter of the dimensionless unit hydrograph would appear to be its second moment of area. Its first moment would of course be unity. The second moment, however, would be equal to the ratio of the second moment of the instantaneous unit hydrograph to the square of its first moment. This quantity can also be measured without deriving the actual instantaneous unit hydrograph or the dimensionless unit hydrograph.

62. The second moment of the instantaneous unit hydrograph about the vertical through its centre of area, can be shown to be equal to the difference between the second moments of the storm run-off and the effective rainfall each about the vertical through their respective centres of area.

63. If desired, the ratio of the third moment to the first moment cubed, of the instantaneous unit hydrograph, could be used as a third parameter of the response, and so on.

64. Such a procedure has the double advantage that it is analogous to well

established methods used in statistics to describe frequency distributions, and that the derivation of the actual instantaneous unit hydrograph, with all the practical difficulties which this entails, is avoided.

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The Paper, which was received on 27 January, 1958, is accompanied by six sheets of drawings, from which the Figures in the text have been prepared.

Written discussion on this Paper should be forwarded to reach the Institution before 15 August, 1958, and will be published in or after December 1958. Contributions should not exceed 1,200 words.—Sec.
