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A Geomorphoclimatic Theory of the Instantaneous Unit Hydrograph

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The instantaneous unit hydrograph is conceived as a random function of climate and geomorphology varying with the characteristics of the rainfall excess. The probability density functions of the peak and time to peak of the IUH are analytically derived as functions of the rainfall characteristics and the basin geomorphological parameters. The main characteristics of these pdf's are studied, and a new approach to hydrologic similarity is initiated under the concept of the geomorphoclimatic IUH. For a given set of geomorphologic characteristics and a particular intensity and duration of rainfall, the peak and time to peak of the IUH corresponding to those values can be easily estimated.

GEOMORPHOLOGIC STRUCTURE OF HYDROLOGIC RESPONSE

The infinite form and variety of drainage basins respond to the known basic geomorphologic laws existing in nature. It is to be expected that in the structure of the hydrologic response of a basin a basic order should also exist which reflects the deep symmetry in formal relations between the parts involved in Horton's geomorphologic laws. *Rodríguez-Iturbe and Valdés* [1979] linked, in an analytical manner, the hydrologic response of a catchment represented by the instantaneous unit hydrograph (IUH) with the geomorphologic parameters of a basin. The IUH was interpreted as the frequency distribution of the times of arrival at the outlet of the basin of water particles, given an instantaneous application of a unit volume of excess rainfall uniformly spread over the catchment at zero time.

The time history of one drop of effective rainfall was viewed as a continuous Markovian process where the state is the order of the stream in which the drop is located at time t. An artificial trapping state was considered to exist at the outlet, and the final goal was to derive the state probability for the trapping state. Figure 1 shows the conceptual scheme for a third-order basin. A drop may land in one of the three states corresponding to areas draining directly into streams of order 1, 2, or 3. The initial state probability is then expressed in terms of geomorphologic parameters as well as the transition probabilities p_{ij} which describe the chances of one drop going from a stream of order i to another stream of order j. The random variable describing the waiting time in a stream of order i is assumed to follow an exponential distribution with parameter λ_i. This assumption is equivalent to that of a linear reservoir, and the exponential response coming from the highest-order stream will produce a hydrograph for the whole basin which does not start at zero. The IUH will start at an ordinate equal to the ordinate at the origin of the partial unit impulse response function corresponding to the highest-order subbasin. For this reason the subbasin of the highest order is artificially represented as two linear reservoirs. State 3*a* in Figure 1 receives the drops from all second-order streams, part of first-order streams, and those drops draining directly into the third-order stream. All these drops are passed to state 3*b*, which is the one that feeds the trapping state.

The IUH is the probability density function for the time of arrival of a randomly chosen drop to the trapping state. The value at the mode of the above pdf produces the main characteristics of the geomorphologic IUH, which are its peak and time to peak, given by *Rodríguez-Iturbe and Valdés* [1979] as

$$q_p = \frac{1.31}{L_\Omega} R_L^{0.43} v \tag{1}$$

$$t_p = \frac{0.44 L_\Omega}{v} \left(\frac{R_B}{R_A}\right)^{0.55} R_L^{-0.38} \tag{2}$$

where R_B, R_A, and R_L are Horton's bifurcation ratio, area ratio, and length ratio, respectively; L_Ω is the length in kilometers of the highest-order stream; and v is the peak velocity of the response in meters per second. Parameters t_p and q_p are given in hours and in per hour, respectively. The units of q_p are $(\text{time})^{-1}$, so that multiplication by area of the basin and depth of effective rainfall produces discharge in $(\text{volume}) \times (\text{time})^{-1}$. Moreover, the analysis of *Rodríguez-Iturbe et al.* [1979] also suggests a most probable value of R_B/R_A of 0.80. The geomorphologic IUH was compared with IUH's derived from the discharge hydrographs produced in controlled experiments based on rainfall-runoff models of several real world basins. The comparison, using different kinds of storms, was remarkably good in all cases [*Valdés et al.*, 1979].

Using the same basin framework described before and working under a kinetic theoretic point of view, *Gupta et al.*

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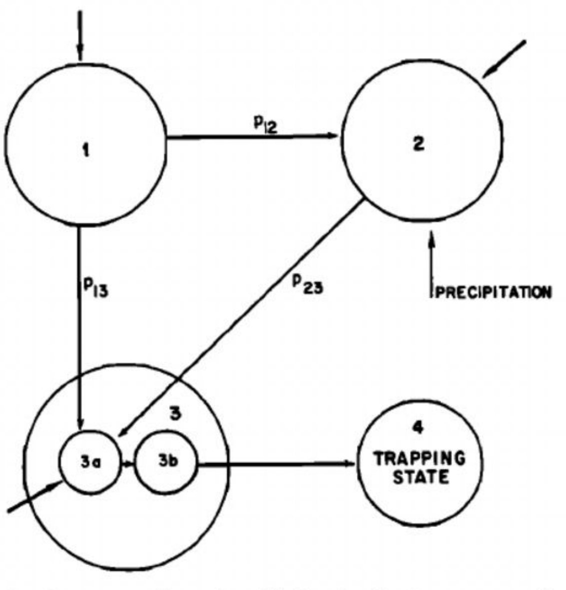

Fig. 1. Representation of a third-order basin as a continuous Markovian process.

[1980] were able to work with any distribution for the waiting time mechanism as well as to get rid of the Markovian hypothesis.

RAINFALL INPUT AND FLOW VELOCITY

An effective rainfall of certain intensity, i_r, that is constant throughout the duration t_r is assumed to occur with a uniform spatial pattern over a basin. The storm intensity i_r and duration t_r are reflected in the velocity parameter v of (1) and (2). Equations (1) and (2) work under the assumption, justified by several other investigators [e.g., *Pilgrim,* 1977], that the flow velocity at a given time during the uniform storm can be taken as reasonably constant throughout the basin. Under the above assumption, v can then be expressed analytically as a function of i_r, t_r, and the geomorphologic characteristics of the average first-order basin, which are themselves related to higher-order basins through the geomorphological laws.

Following the hypothesis of the geomorphologic IUH [*Rodríguez-Iturbe and Valdés,* 1979], we will assume that overland travel time may be neglected and allowance will be made only for stream travel time. Figure 2 shows the average first-order subcatchment with concentration time [*Eagleson,* 1970]

$$t_c = \left(\frac{\bar{L}_1 i_*^{1-m_s}}{\alpha_s} \right)^{1/m_s} \tag{3}$$

where

$$i_* = \frac{\bar{A}_1}{\bar{L}_1} i_r \tag{4}$$

Here α_s and m_s are the kinematic wave parameters of the average first-order channel ($m_s = 5/3$), $\bar{A}_1$ and $\bar{L}_1$ represent the average area and the average stream length of first-order subbasins, i_r is the effective rainfall spread uniformly over the idealized average first-order subcatchment, and i_* is the lateral inflow per unit length of stream. $\bar{L}_0$ and α_0 in Figure 2 refer to the width of the basin and the kinematic wave parameter of the overland flow, respectively.

The maximum discharge in Figure 2 depends on the relative duration of the rainfall with respect to the concentration time of the stream

Case A

$$t_r < t_c \qquad q_{\max} = \alpha_s (i_* t_r)^{m_s} \tag{5}$$

Case B

$$t_r > t_c \qquad q_{\max} = \bar{L}_1 \cdot i_* \tag{6}$$

The peak velocity is given by

$$v_{\max} = \alpha_s^{1/m_s} q_{\max}^{(m_s-1)/m_s} \tag{7}$$

thus

Case A

$$t_r < t_c \qquad v_{\max} = \alpha_s (i_* t_r)^{m_s-1} \tag{8}$$

Case B

$$t_r > t_c \qquad v_{\max} = \alpha_s^{1/m_s} (\bar{L}_1 \cdot i_*)^{(m_s-1)/m_s} \tag{9}$$

THE GEOMORPHOCLIMATIC IUH

Since i_r and t_r are random variables whose distributions represent the influence of climate, we propose the reinterpretation of the IUH as a stochastic unit impulse response function. The peak and time to peak, q_p and t_p, of the geomorphoclimatic IUH are random variables whose distributions depend on the geomorphology of the basin and on the climate specified through the distribution of i_r and t_r. Our goal in this section is to derive the pdf's for q_p and t_p and provide in this manner a geomorphoclimatic framework to the general theory of the IUH presented by *Dooge* [1965].

It is important to point out that (8) and (9) when used to describe the velocity in (1) and (2) for q_p and t_p of the geomorphologic IUH imply that the response function depends on the intensity and duration of the rainfall excess. Thus we are working in a nonlinear framework which is at variance with the traditional linear unit hydrograph theory. Since each stream order is described by an average channel with constant α_s—meaning constant cross section and roughness—one could argue that real basins may approach linearity at high flows as a result of greater roughness on the

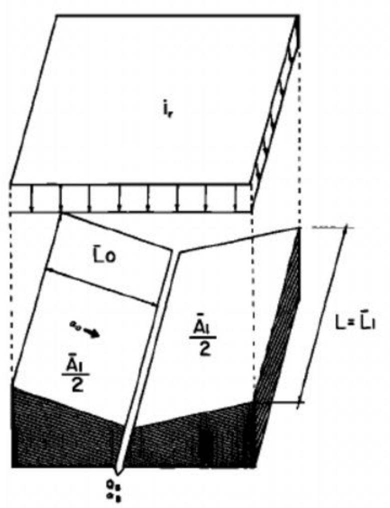

Fig. 2. Schematic of the average first-order subcatchment.

high banks and changing channel cross sections. This is something that needs research with good historical data, although there exist many examples in the hydrologic literature where the response function has been shown to depend on the rainfall input [e.g., *Minshall*, 1960].

Distribution of q_p

From (1) we can write the pdf of q_p as

$$f(q_p) = \left(\frac{1}{\theta}\right) f_V\left(\frac{q_p}{\theta}\right) \quad (10)$$

where

$$\theta = \frac{1310 R_L^{0.43}}{L_\Omega} \quad (11)$$

with L_Ω now in meters.

Term $f_V(q_p/\theta)$ in (10) stands for the pdf of the peak velocity v in the right-hand side of (1), and the argument of the function is q_p/θ instead of v. Factor $(1/\theta)$ is the Jacobian of the transformation. The problem is now to derive $f_V(v)$:

$$f_V(v) = \omega f_V^{(1)}(v) + (1-\omega) f_V^{(2)}(v) \quad (12)$$

where $\omega = \text{Prob } t_r < t_c$, $f_V^{(1)}(v)$ stands for the case $t_r < t_c$ and $f_V^{(2)}(v)$ for the case $t_r > t_c$.

Case $t_r < t_c$

From (8) we have

$$v = kh^\xi \quad (13)$$

where

$$k = \left(\frac{\bar{A}_1}{\bar{L}_1}\right)^{m_s - 1} \cdot \alpha_s \quad (14)$$

$$\xi = m_s - 1 \quad (15)$$

$$h = i_r t_r \quad (16)$$

From derived distribution analysis we have

$$f_V(v) = \frac{dh}{dv} f_H(h) \quad (17)$$

Thus we may write

$$f_V^{(1)}(v) = \frac{1}{k\xi}\left(\frac{v}{k}\right)^{(1-\xi)/\xi} f_H\left[\left(\frac{v}{k}\right)^{1/\xi}\right] \quad (18)$$

where $f_H(\ )$ stands for the pdf of the random variable h. The argument of $f_H(\ )$ is written as function of v using (13). The intensity and duration of effective rainfall are now assumed to be independent random variables with exponential distributions, a hypothesis that appears to be justified at least for events beyond a certain threshold:

$$f_{T_R}(t_r) = \delta e^{-\delta t_r} \quad (19)$$

$$f_{I_R}(i_r) = \beta e^{-\beta i_r} \quad (20)$$

Equations (19) and (20) for the intensity and duration of effective rainfall have been used extensively for different types of climates and seem to provide a satisfactory description of the variables [*Eagleson*, 1970, 1982].

The distribution of h is then given by *Eagleson* [1982] as

$$f_H(h) = 2\beta\delta K_0[2(\beta\delta h)^{1/2}] \quad (21)$$

where $K_0(\ )$ is the modified Bessel function of order zero. Substituting (21) into (18) and using the expressions

$$\bar{L}_1 = L_\Omega R_L^{1-\Omega} \qquad \bar{A}_1 = A_\Omega R_A^{1-\Omega}$$

one gets

$$f_V^{(1)}(v) = \frac{2\beta\delta L_\Omega R_L^{1-\Omega}}{(m_s-1)\alpha_s A_\Omega R_A^{1-\Omega}}\left(\frac{v}{\alpha_s}\right)^{(2-m_s)/(m_s-1)} \cdot K_0\left[2\left(\frac{\beta\delta L_\Omega R_L^{1-\Omega}}{A_\Omega R_A^{1-\Omega}}\right)^{1/2}\left(\frac{v}{\alpha_s}\right)^{1/2(m_s-1)}\right] \quad (22)$$

Case $t_r > t_c$

From (9) we have

$$v = k i_r^\xi \quad (23)$$

where now

$$k = \alpha_s^{1/m_s} \bar{A}_1^{(m_s-1)/m_s} \qquad \xi = \frac{m_s - 1}{m_s} \quad (24)$$

Then

$$f_V^{(2)}(v) = \frac{1}{k\xi}\left(\frac{v}{k}\right)^{(1-\xi)/\xi} f_{I_R}[(v/k)^{1/\xi}] \quad (25)$$

which after some operations becomes

$$f_V^{(2)}(v) = \frac{m_s \beta}{(m_s-1)A_\Omega R_A^{1-\Omega}}\left(\frac{v}{\alpha_s}\right)^{1/(m_s-1)} \cdot \exp\left[\frac{-\beta v^{m_s/(m_s-1)}}{A_\Omega R_A^{1-\Omega}\alpha_s^{1/(m_s-1)}}\right] \quad (26)$$

Having now $f_V^{(1)}(v)$ and $f_V^{(2)}(v)$, the simple application of (10) yields $f^{(1)}(q_p)$ $f^{(2)}(q_p)$, which are then weighted by ω and $(1-\omega)$ in order to obtain the pdf of the peak of the IUH, $f(q_p)$.

Prob $(t_r < t_c)$

The probability, ω, that t_r is less than the concentration time of the average first-order stream may be written in a conditional form as

$$\omega | i_r = \int_0^{t_c(i_r)} \delta e^{-\delta t_r}\, dt_r = 1 - \exp\left[\frac{-\delta \bar{L}_1 (i_r \bar{A}_1)^{(1-m_s)/m_s}}{\alpha_s^{1/m_s}}\right] \quad (27)$$

where use has been made of (3) and (4). We now remove the conditioning on i_r,

$$\omega = \int_0^\infty \left[1 - \exp\left(\frac{-\delta \bar{L}_1 (i_r \bar{A}_1)^{(1-m_s)/m_s}}{\alpha_s^{1/m_s}}\right)\right] \beta e^{-\beta i_r}\, di_r \quad (28)$$

With $m_s = 5/3$ one gets

$$\omega = 1 - \beta \int_0^\infty \exp\left(-\beta i_r - \frac{\delta \bar{L}_1}{\alpha_s^{0.6}} \bar{A}_1^{-0.4} i_r^{-0.4}\right) di_r \quad (29)$$

The integral in (29) may be written as

$$I(i_r) = \int_0^\infty e^{-(bi_r + c/i_r^{0.4})}\, di_r \quad (30)$$

with $b = \beta$ and $c = \delta \bar{L}_1 \bar{A}_1^{-0.4}/\alpha_s^{0.6}$. $I(i_r)$ is the same kind of integral that was evaluated by *Eagleson* [1982] in an approximate manner. Following the same procedure of *Eagleson* [1982], one gets after some manipulation

$$I(i_r) = \frac{e^{-2\sigma}\Gamma(\sigma + 1)}{\beta \sigma^\sigma} \quad (31)$$

where

$$\sigma = \beta \cdot \left(\frac{0.4\delta \bar{L}_1 \bar{A}_1^{-0.4}}{\beta \alpha_s^{0.6}} \right)^{5/7} \quad (32)$$

Now σ is written as

$$\sigma = \beta^{2/7}(0.4\delta)^{5/7}(A_\Omega R_A^{1-\Omega})^{-2/7}(L_\Omega R_L^{1-\Omega})^{5/7}\alpha_s^{-3/7} \quad (33)$$

The probability that $t_r < t_c$ is from (29)

$$\omega = 1 - \frac{e^{-2\sigma}\Gamma(\sigma + 1)}{\sigma^\sigma} \quad (34)$$

where σ is given by (33) and $\Gamma(\ )$ stands for the gamma function.

The distribution of the peak of the geomorphoclimatic IUH is then

$$f(q_p) = \omega f^{(1)}(q_p) + (1 - \omega) f^{(2)}(q_p) \quad (35)$$

where ω is given by (34) and $f^{(1)}(q_p)$ and $f^{(2)}(q_p)$ are given by (10), $f_V(q_p/\theta)$ being defined through (22) and (26).

Distribution of t_p

From (2), with $R_B/R_A = 0.80$, we can write the pdf of t_p as

$$f(t_p) = \frac{K}{t_p^2} f_V\left(\frac{K}{t_p}\right) \quad (36)$$

where

$$K = \frac{3.9 \times 10^{-4} L_\Omega}{R_L^{0.38}} \quad (37)$$

L_Ω being in meters and t_p still in hours.

Following the same procedure that we employed to obtain $f(q_p)$, the distribution of the time to peak of the geomorphoclimatic IUH may be written as

$$f(t_p) = \omega f^{(1)}(t_p) + (1 - \omega) f^{(2)}(t_p) \quad (38)$$

where ω is defined by (34) and $f^{(1)}(t_p)$ and $f^{(2)}(t_p)$ are given by (36), with $f_V(K/t_p)$ being defined through (22) and (26).

CHARACTERISTICS OF THE GEOMORPHOCLIMATIC IUH

The probability distributions of q_p and t_p which contain the essence of the geomorphoclimatic IUH are made up of the weighted sum of two distributions in each case. The weight is the probability that the rainfall duration is below or above the concentration time of the average first-order stream. Intuitively, one would think that ω in (35) and (38) is very close to zero. To study the order of magnitude of ω, one may consider a relatively large basin coupled to a climate of short and intense rainfalls. In this case we are trying to maximize $\omega = \text{Prob}\ t_r < t_c$ by trying to couple a small value of t_r with a large value of t_c as given by (3).

Thus let us fix

$$\bar{i}_r = 2\ \text{cm/h} \qquad \bar{t}_r = 30\ \text{min} \qquad A_\Omega = 400\ \text{km}^2$$

$$R_A = 4 \qquad \Omega = 7 \qquad L_\Omega = 20\ \text{km} \qquad R_L = 2.5$$

The value of σ given by (33) is in this case

$$\sigma = 0.068 \times \alpha_s^{-3/7} \quad (39)$$

where α_s is the kinematic wave parameter for the average first-order stream.

For a rectangular channel, α_s is defined as

$$\alpha_s = \frac{S^{1/2}}{nb^{2/3}} \quad (40)$$

where S is the slope, b the width of the channel, and n the Manning roughness coefficient. Let us assume values which will produce a large concentration time in the stream,

$$S = 0.01 \qquad n = 0.10 \qquad b = 10\ \text{m}$$

In this case, $\alpha_s = 0.215$, quite a small value for a first-order stream. Using this value of α_s in (39), one obtains $\sigma = 0.131$, which, used in (34), yields $\omega = 0.057$. Even in this case where we have imposed conditions leading to a large probability that $t_r < t_c$, ω is less than 6%.

The following approximations are then justified

$$f(q_p) = f^{(2)}(q_p) \quad (41)$$

$$f(t_p) = f^{(2)}(t_p) \quad (42)$$

which means that

$$f(q_p) = \frac{m_s \beta}{\theta(m_s - 1) A_\Omega R_A^{1-\Omega}} \left(\frac{q_p}{\theta \alpha_s}\right)^{1/(m_s-1)} \cdot \exp\left[\frac{-\beta}{A_\Omega R_A^{1-\Omega} \alpha_s^{1/(m_s-1)}} \left(\frac{q_p}{\theta}\right)^{m_s/(m_s-1)}\right] \quad (43)$$

$$f(t_p) = \frac{K m_s \beta}{t_p^2 (m_s - 1) A_\Omega R_A^{1-\Omega}} \left(\frac{K}{t_p \alpha_s}\right)^{1/(m_s-1)} \cdot \exp\left[\frac{-\beta}{A_\Omega R_A^{1-\Omega} \alpha_s^{1/(m_s-1)}} \left(\frac{K}{t_p}\right)^{m_s/(m_s-1)}\right] \quad (44)$$

Parameters θ and K have been defined in (11) and (37), respectively. Using $m_s = 5/3$ and the expressions for θ and K, we may write $f(q_p)$ and $f(t_p)$ as

$$f(q_p) = \frac{3.534 L_\Omega}{\bar{i}_r R_L^{0.43} R_A^{1-\Omega} A_\Omega} \cdot \left(\frac{q_p L_\Omega}{\alpha_s R_L^{0.43}}\right)^{1.5} \cdot \exp\left[\frac{-1.412}{\bar{i}_r A_\Omega R_A^{1-\Omega} \alpha_s^{1.5}} \left(\frac{L_\Omega q_p}{R_L^{0.43}}\right)^{2.5}\right] \quad (45)$$

$$f(t_p) = \frac{0.656 L_\Omega}{\bar{i}_r R_L^{0.38} t_p^2 A_\Omega R_A^{1-\Omega}} \cdot \left(\frac{L_\Omega}{R_L^{0.38} t_p \alpha_s}\right)^{1.5}$$

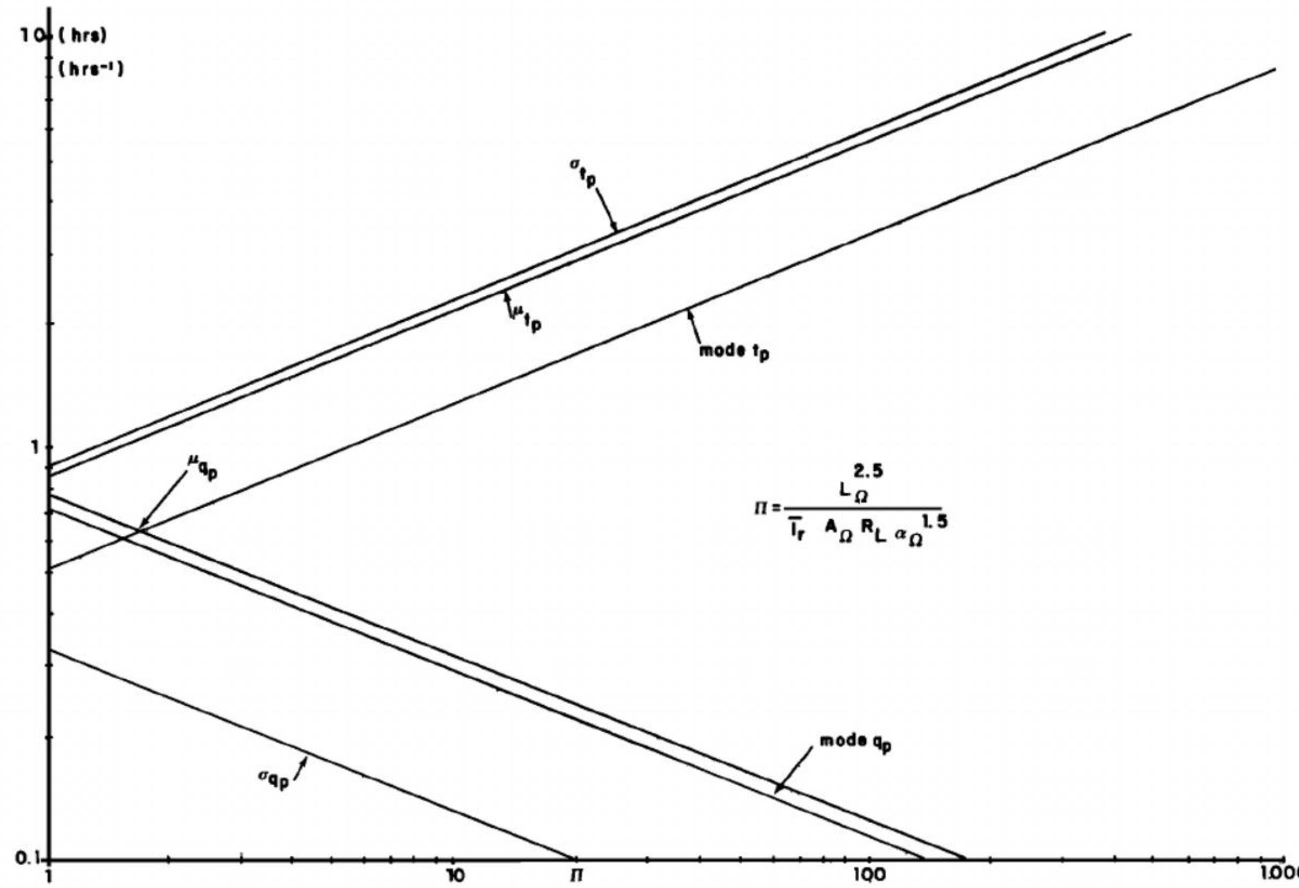

Fig. 3. Mean, standard deviation, and mode of the geomorphoclimatic distributions $f(q_p)$ and $f(t_p)$ as function of Π.

$$\cdot \exp\left[\frac{-0.262}{\bar{i}_r\alpha_s^{1.5}A_\Omega R_A^{1-\Omega}}\left(\frac{L_\Omega}{R_L^{0.38}t_p}\right)^{2.5}\right] \quad (46)$$

where we have switched to the more common units of A_Ω (square kilometers), L_Ω (kilometers), $\bar{i}_r$ (centimeters per hour), and α_s (s^{-1} $m^{-1/3}$). The units of α_s are the ones resulting from its estimation through (40), the width, b, being in meters and the Manning roughness, n, in s/m$^{1/3}$. It is worth recalling that n values used in practice are the same in metric and in English systems, unit conversion being accomplished by the factor 1.49, which Manning's equation carries in the English system. The q_p and t_p in (45) and (46) remain in per hour and hours, respectively.

From the assumption of constant velocity throughout the channel network one may then consider a storm lasting longer than the concentration time of the whole basin for which

$$v_1 = \alpha_1^{1/m_s}Q_1^{(m_s-1)/m_s} = \alpha_\Omega^{1/m_s}Q_\Omega^{(m_s-1)/m_s} = v_\Omega \quad (47)$$

Here Q_1 and Q_Ω are the peak discharges in the basins of order 1 and Ω, respectively; α_1 and α_Ω are kinematic wave parameters for the average channel of order 1 and the highest-order channel, respectively. Since we are dealing with effective rainfall,

$$Q_\Omega = i_r A_\Omega \qquad Q_1 = i_r\bar{A}_1 \quad (48)$$

and thus

$$\alpha_1 = \alpha_\Omega(R_A^{\Omega-1})^{m_s-1} \quad (49)$$

Equation (49) can be substituted for α_s in (45) and (46) with $m_s = 5/3$. The result is that the term $R_A^{1-\Omega}$ disappears and α_s becomes α_Ω, which is much easier to estimate. In this manner, $f(q_p)$ and $f(t_p)$ are independent of the subjective determination of the order Ω of the basin.

For the range of values which R_L takes in nature, one may assume $R_L^{0.43} \simeq R_L^{0.38} \simeq R_L^{0.40}$. If we now call

$$\Pi = \frac{L_\Omega^{2.5}}{\bar{i}_r A_\Omega R_L \alpha_\Omega^{1.5}} \quad (50)$$

(45) and (46) can then be written

$$f(q_p) = 3.534\Pi q_p^{1.5}\cdot\exp(-1.412\Pi q_p^{2.5}) \quad (51)$$

$$f(t_p) = \frac{0.656\Pi}{t_p^{3.5}}\cdot\exp\left(\frac{-0.262\Pi}{t_p^{2.5}}\right) \quad (52)$$

Equations (51) and (52) represent the pdf's of the peak and time to peak of the geomorphoclimatic IUH. Only one parameter, Π, controls both distributions. Π given by (50) is a function of climate and geomorphology and plays a key role in the IUH properties.

Moments of $f(q_p)$ and $f(t_p)$

The moments of $f(q_p)$ and $f(t_p)$ can be estimated through the integral [*Gradshteyn and Ryzhik*, 1965]:

$$\int_0^\infty x^{\nu-1}e^{-\mu x}\,dx = \frac{\Gamma(\nu)}{\mu^\nu} \quad (53)$$

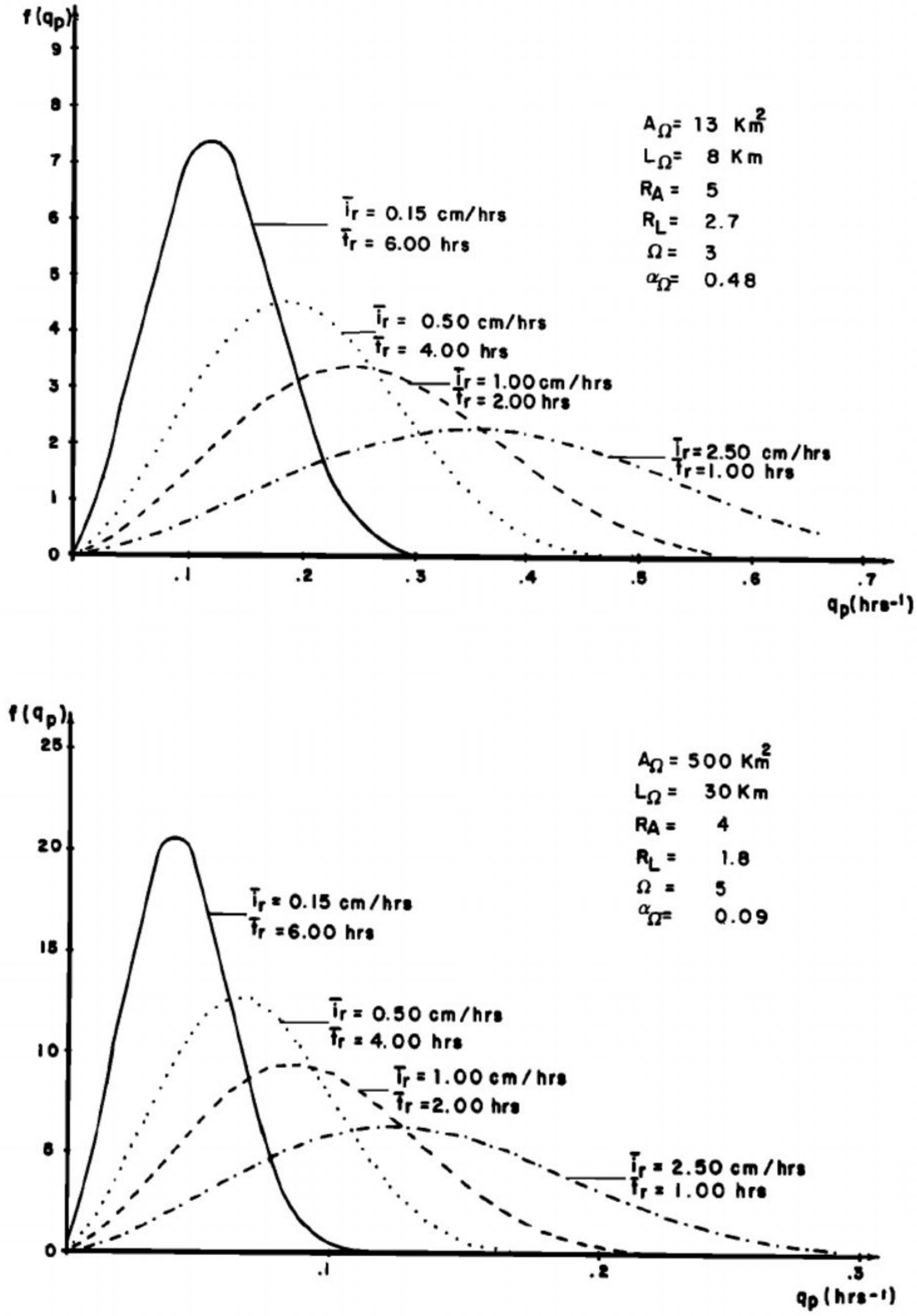

Fig. 4. Examples of the probability density function of the peak of the IUH.

We thus have after some manipulation:

$$E(q_p) = \frac{0.774}{\Pi^{0.4}} \qquad \sigma_{q_p} = \frac{0.327}{\Pi^{0.4}} \tag{54}$$

$$C_V(q_p) = 0.423 \qquad \text{mode } q_p = \frac{0.710}{\Pi^{0.4}}$$

$$E(t_p) = 0.858\Pi^{0.4} \qquad \sigma_{t_p} = 0.915\Pi^{0.4}$$

$$C_V(t_p) = 1.066 \qquad \text{mode } t_p = 0.512\Pi^{0.4} \tag{55}$$

The mean, mode, and standard deviation for q_p and t_p as function of Π are shown in Figure 3, where they are represented by straight lines on a logarithmic scale.

Figures 4 and 5 show examples of $f(q_p)$ and $f(t_p)$ for two geomorphological configurations and four different climates. It is interesting to remark that the variance of $f(q_p)$ is much smaller for low rainfall intensities than for larger intensities of precipitation. The opposite behavior is found in the variance of $f(t_p)$. Nevertheless, the coefficient of variation is constant in both distributions independent of climate and geomorphology. These examples show how much variability

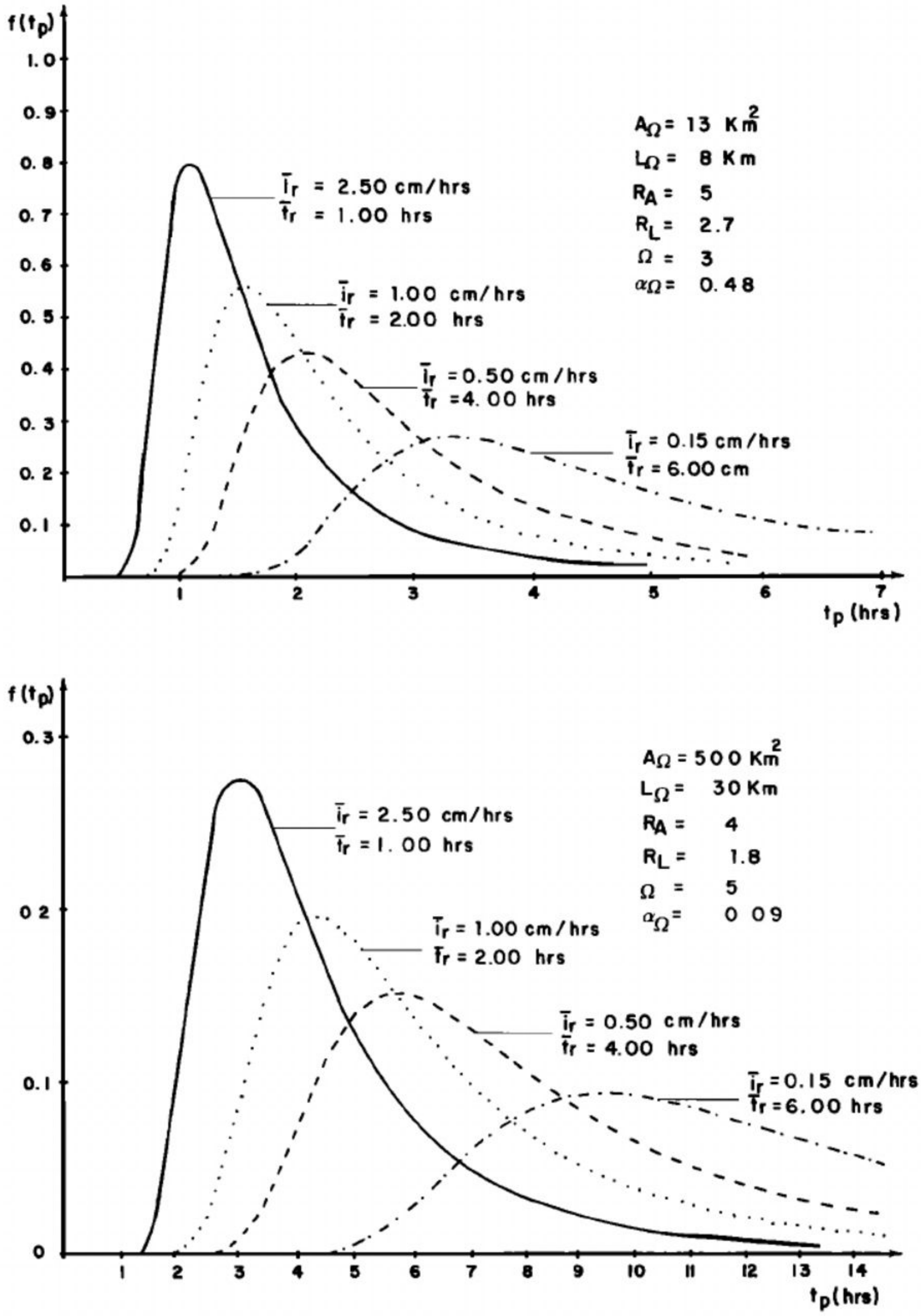

Fig. 5. Examples of the probability density function of the time to peak of the IUH.

in the characteristics of the IUH can be found through (51) and (52) when all the events of a particular climate are considered for potential estimation of the response function of a basin. From a different perspective, they also show the error that can be made when convoluting a storm with an IUH derived from an event with very different rainfall characteristics. Again we will point out that analytical approaches as the one taken here or schemes based on rainfall-runoff modeling may somewhat increase the nonlinearity of the response and a real basin could conceivably approach a more linear behavior at high flows as a result of the changes in roughness and channel cross sections.

The estimation of parameter Π does not present complications; nevertheless, its value is quite sensitive to α_Ω (equation (50)). Figures 6 and 7 show examples of the dependence of $f(q_p)$ and $f(t_p)$ on α_Ω for a certain climate and two geomorphological configurations. The sensitivity of the geomorphoclimatic distributions to α_Ω is expected, since it is through $\bar{i}_r$ and α_Ω that the dynamic characteristics of the system are reflected in the analysis.

IUH for a Particular Storm

From (23) one may write for the peak velocity

$$v = \alpha_s^{1/m_s}(i_r\bar{A}_1)^{(m_s-1)/m_s} \qquad (56)$$

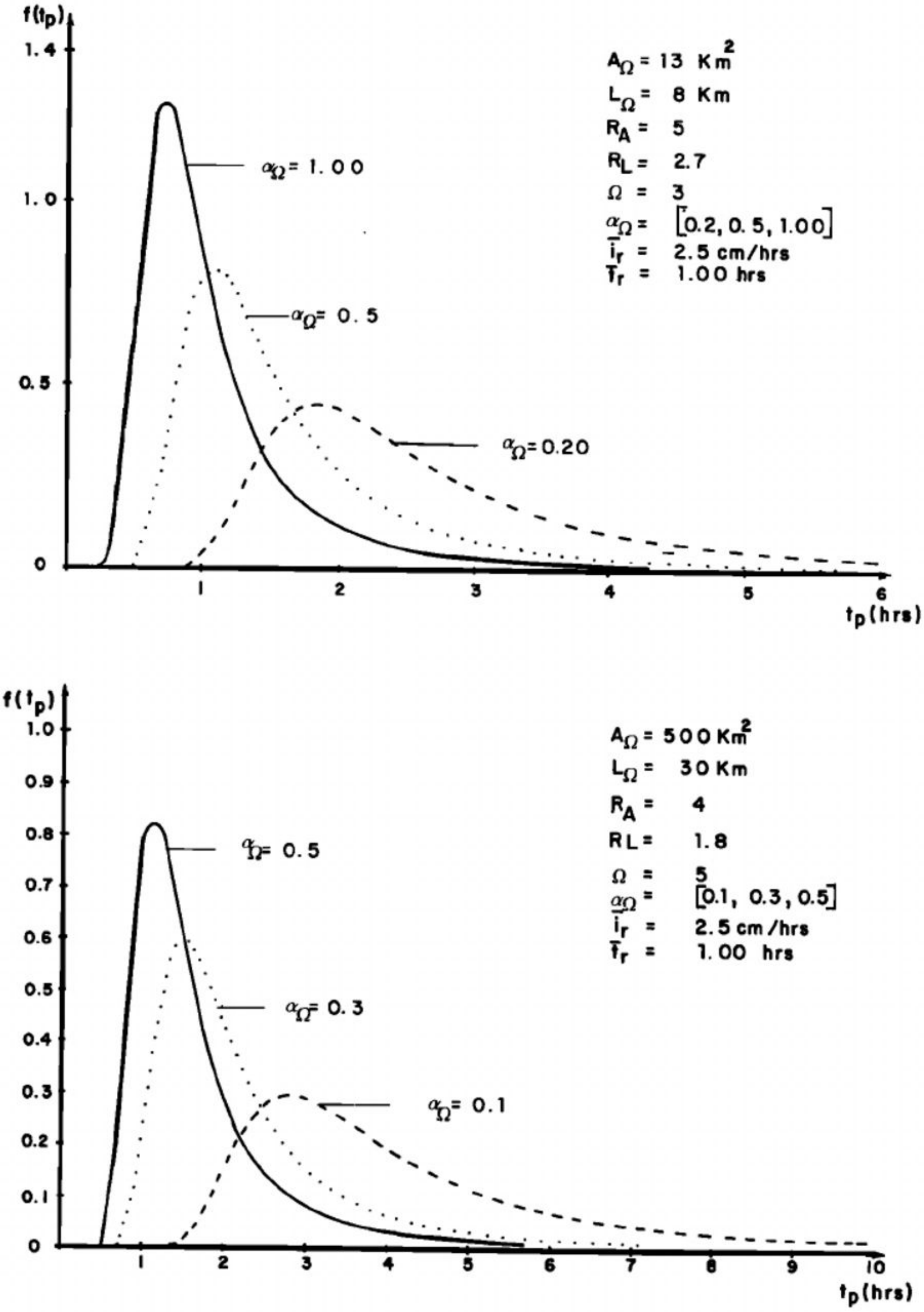

Fig. 6. Sensitivity of $f(q_p)$ to values of α_Ω.

Using (49) for α_s, taking $m_s = 5/3$ and $\bar{A}_1 = A_\Omega R_A{}^{1-\Omega}$, one then has

$$v = 0.665\alpha_\Omega{}^{0.6}(i_r A_\Omega)^{0.4} \quad (57)$$

where the coefficient takes care of the conversion in units. A_Ω is in square kilometers, i_r in centimeters per hour, v in meters per second, and α_Ω, as before, in $s^{-1}\ m^{-1/3}$.

Equation (57) can be substituted for v in (1) to give the estimate of q_p for a particular storm i_r. Similarly, it can be used in (2) with $R_B/R_A = 0.80$ to give the estimate of t_p:

$$q_p = \frac{0.871}{\Pi_i{}^{0.4}} \quad (58)$$

$$t_p = 0.585\Pi_i{}^{0.4} \quad (59)$$

where

$$\Pi_i = \frac{L_\Omega{}^{2.5}}{i_r A_\Omega R_L \alpha_\Omega{}^{1.5}} \quad (60)$$

Notice that the expression for Π_i is the same as that for Π (equation (50)) except that $\bar{i}_r$ becomes now i_r.

The expressions for q_p and t_p given in (58) and (59), coupled with the assumption of a triangular IUH, allow for the estimation of the peak and time to peak of the discharge response to any given event of a certain intensity and

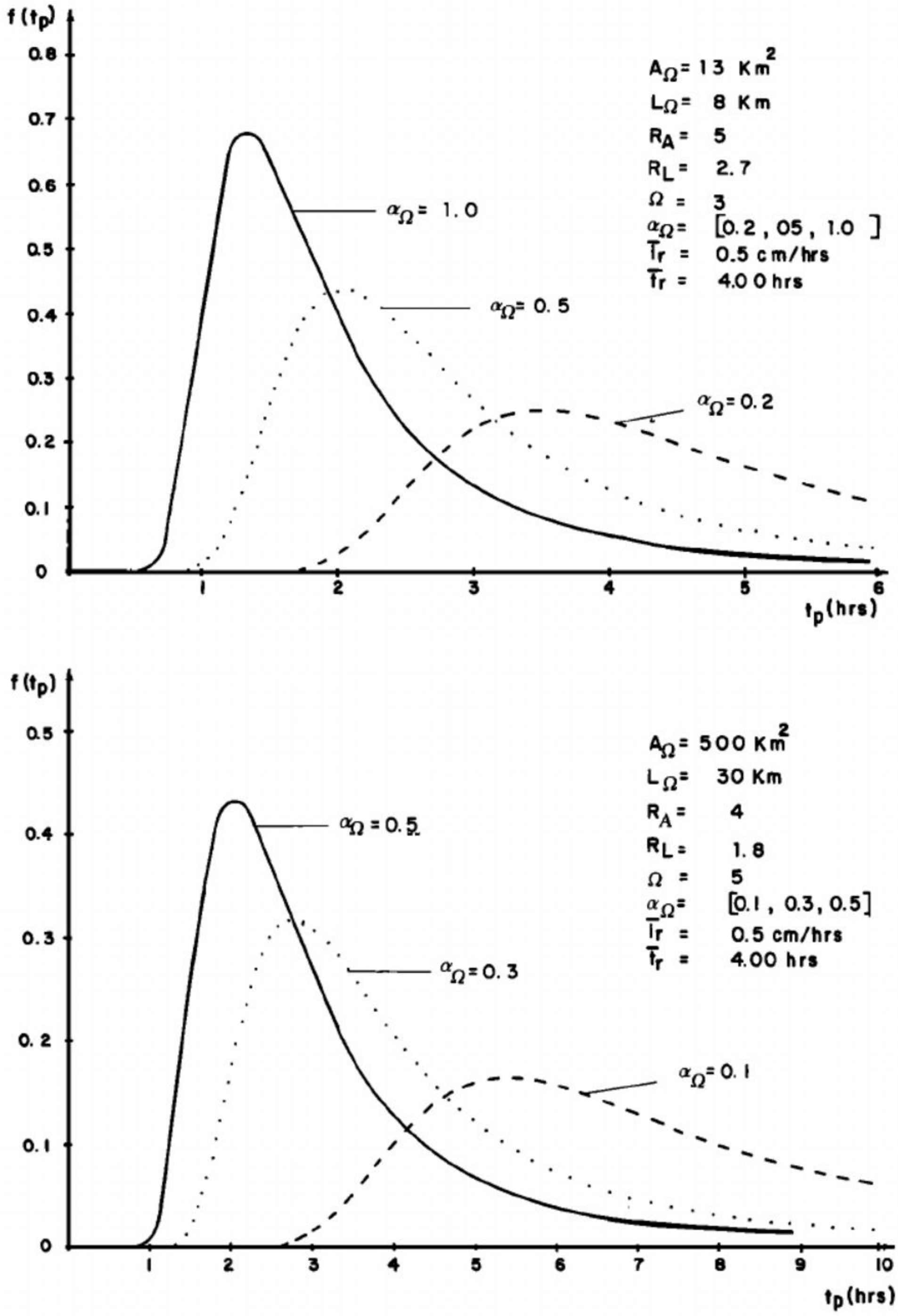

Fig. 7. Sensitivity of $f(t_p)$ to values of α_Ω.

duration of effective rainfall [*Rodríguez-Iturbe et al.*, this issue]. Moreover, this kind of analysis opens an avenue for a simple and approximate estimation of flood probabilities when one uses total rainfall and catchment characteristics in the parameter Π_i on the one hand and some soil constants on the other [*Córdova and Rodriguez-Iturbe,* 1982].

Final Comments

A link has been established between climate, the geomorphologic structure, and the hydrologic response of a basin. This link explains in a quantitative manner the well-known variations which hydrologists find when estimating IUH's from different storms in the same basin. The geomorphoclimatic IUH allows the estimation of the unit impulse response function for a given particular rainfall input which the hydrologist may wish to consider. Moreover, this estimation is carried out independent of any climatic data for particular rainfall-runoff events. Since the data traditionally used in IUH derivations are very much noise corrupted in both rainfall and runoff and, moreover, since the treatment of the data is also highly subjective (e.g., effective rainfall estimation, uniform spatial distribution, etc.), the use of the geomorphoclimatic IUH may well be more recommendable than direct estimation from real data. Besides the problems related to the data, in the classical unit hydrograph analysis the hydrologist confronts the unavoidable fact that he will be using an IUH derived from a particular storm to forecast the output of a storm different from the first one. The geomorphoclimatic theory of the IUH quantitatively gives the serious risks involved in the above procedure; even more important than that, we believe that it is a step toward the understanding of hydrologic processes at a basin scale, since to explain the world of hydrologic phenomena, it will be necessary to develop scientific theories of a general character. The basic philosophy of this research lies in the belief that hydrologic response at the basin scale depends only on some of the gross features of the basin and not on the details of the network geometry. Up to now the channel network and contributing areas have been described through Horton relations and Strahler ordering procedures. The results are encouraging, but it is important to point out that some very recent research by *Gupta and Waymire* [1982] has shown that Strahler ordering may not be the most adequate way to describe network geometry when studying hydrologic response. A richer avenue from a theoretical point of view may lie in the description of the network through the density of link occurrences at a different level of space. This approach, initiated by *Gupta and Waymire* [1982], is the subject of an ongoing joint research project by the University of Mississippi and Universidad Simón Bolívar.

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