# CURVATURE EFFECTS ON HYDRAULIC INSTABILITY

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## INTRODUCTION

With the information available on the various manifestations of instability in a continuous medium, three main categories con be formulated. These are, first, the oscillations of parallel flows, or nearly parallel ( quasi-parallel ) flows such as - channel flows and boundary layers. Second, there is the class of flows with curved streamlines, such as vortices between rotating cylinders or boundary layers along curved walls. In the third category are those cases where the mean flow is truly zero, such as Bénard cells and convective instabilities. Brock (1) observed that "when water flows down a long sufficiently steep open channel, it is found that the depth of flow is not as uniform as it would be if the same channel had a very small slope. The flow is characterized by a series of hydraulic bores that extend across the width of the channel and propagate downstream. Across these bores or shocks the depth of flow varies abruptly. Between successive bores the depth of flow varies gradually , Waves of this kind are termed roll waves and flows with such waves are called slug flows by some workers".

Many analyses have been made concerning this phenomenon, but invariably, they considered the channel bottom linear. It is the aim of this paper to analyze hydrodynamic free surface instability resulting from the combined action of resistance and curvature of the channel floor by using the generalized nonlinear shallow-flow equations recently developed by Dressler (8).

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## PREVIOUS WORK

This section focuses on previous works on the flow instability mechanism on - open channels.

The phenomenon of free surface instability was first examinen by Jeffreys (10) who considered flow in a wide rectangular channel with uniform velocity distribution and Chézy friction formula, and concluded that the condition for instability was that $F > 2$, where $F$ is the Froude number.

Keulegan and Patterson (11) determined mathematically a stability criterion - from Boussinesq's equation for wave celerity and the Manning formula. Their stability criterion is

$$S_o - S_f \begin{bmatrix}\text{unstable}\\ \text{stable}\end{bmatrix} \gtrless 0 \qquad (1)$$

in which $S_o$ and $S_f$ are the bed and friction slope, respectively.

Dressler (5) and Thomas (16) performed mathematical analyses of several aspects of the surge motion. Using the shock energy inequality to construct actual roll wave solutions, Dressler (5) observed that such solutions could never be constructed if the variation of resistance effect with depth was neglected.

Vedernikov (17, 18) generalized Jeffreys findings for a channel of arbitrary cross-sectional shape and determined a stability criterion by using certain approximations to the Saint Venant equations. His criterion is

$$V = \left(\frac{1-b}{2+b}\right)\left(1 - R\frac{dP}{dA}\right) F \qquad (2)$$

in which $V$ is the Vedernikov number, $b$ is the coefficient in the resistance law $f = a\,(R)^b$ for a hydraulically smooth conduit, $R$ is the hydraulic radius, $P$ is the wetted perimeter, $A$ is the cross-sectional area, $F$ is the Froude number and $a$ is a coefficient. The flow is unstable when $V > 1$.

Craya (2) showed that the flow becomes unstable when the Seddon celerity, $dQ/dA$ is greater than the Lagrangian celerity, $v + [gy]^{1/2}$, i.e.,

$$dQ/dA > v + [gy]^{1/2} \qquad (3)$$

where $Q$ is the discharge, $A$ the cross-sectional area, $v$ is the average velocity, $g$ is the gravitational acceleration and $y$ is the flow depth.

Dressler and Pohle (6) adapted the method proposed by Jeffreys (10) to a gene ral resistance function and derived

$$U > \frac{q}{m}\left[g \;Y\; \cos\theta\right]^{1/2} \tag{4}$$

which is identical with the Vedernikov (17, 18) and Craya (2) results obtained by other methods under somewhat different assumptions. In Eq. (4), U and Y are the velocity and water depth for linear uniform flow respectively; g is the gravi tational acceleration, $\theta$ the angle of inclination of the channel below the horizontal, m and q are exponents in the general resistance function $\lambda u^{q}/y^{m}$ in which u and y are velocity and depth of flow respectively, and $\lambda$ is a measure the roughness.

Dracos and Glenne (4) investigated whether a perturbed water surface along a characteristic tends to steepen or flatten. If it tends to steepen, the perturbation will grow into a breaking wave.

Gradowczyk (9) extended the linear stability analysis of open channel flows to include the erodibility of a noncohesive bed. The critical Froude number $F_c$ above which the free-surface of the fluid becomes unstable in a rigid and flat channel is

$$F_c^2 = \frac{4}{(1+j)^2} \tag{5}$$

where j is a constant and $F_c$ is the critical Froude number.

Ponce and Simons (14) applied the theory of linear stability (13) to the Saint Venant equations (3) which governs the flow motion in open channels. They validated the theoretical and observed fact that roll waves are formed when $F > 2$ ( for wide channels using Chézy friction ).

All the above presented criteria are strictly valid for channels with a straight bottom. To overcome the inherent limitations imposed by the Saint Venant equations (19), Dressler (8) derived expressions that incluide terms containing not only the channel slope $\theta$, but also containing explicitly the bottom curvature k and its rate of change k'.

It is the objective of this paper to carry out a stability analysis using the Dressler generalized nonlinear shallow-flow equations. The methodology used is the one outlined by Dressler and Pohle (6).

## THEORETICAL DEVELOPMENT

In this section, the Dressler generalized nonlinear shallow-flow equations are presented. Linearized perturbation theory is used to obtain the differential equations governing the perturbations, and stability criterion is derived.

Governing Equations.- Following Dressler (11), the nonlinear shallow-flow equations with curvature expressed in a curvilinear coordinate system for nonviscous fluids (Fig. 1) are the continuity equation

$$\frac{\partial N}{\partial t} + \frac{1}{(1-kN)^2} C \frac{\partial N}{\partial s} - \frac{\ln(1-kN)}{(1-kN)k} \frac{\partial C}{\partial s} + \frac{k'}{k^2}\left[\frac{k}{(1-kN)^2} N + \frac{\ln(1-kN)}{(1-kN)}\right] C = 0 \tag{6}$$

and the equation of motion

$$\frac{\partial C}{\partial t} + \frac{1}{(1-kN)^2} C \frac{\partial C}{\partial s} + \left[g \cos\theta + \frac{k}{(1-kN)^3} C^2\right] \frac{\partial N}{\partial s} - \left[kg \sin\theta - \frac{k'}{(1-kN)^3} C^2\right] N + g \sin\theta = 0 \tag{7}$$

for the unknowns $C(s,t)$ and $N(s,t)$.

In Eqs. (6) and (7), $C$ represents the tangential velocity along the channel bottom, $N$ the depth of flow, $s$ the length coordinate measured along the bottom curve, $t$ is time, $\theta$ is the channel floor inclination, $g$ is the gravity acceleration, $k$ is the bottom curvature and $k'$ is the rate-of-change of curvature. The experimentally confirmed validity of the Chézy equation for highly unsteady flows (7) makes the modified Chézy formula

$$- \frac{\lambda C^2}{\rho N}\left[\frac{1}{1 - \frac{kN}{2}}\right] \tag{8}$$

applicable to flows over a curved surface. In the above expression, $\lambda$ is a measure of the roughness and $\rho$ is the density of water.

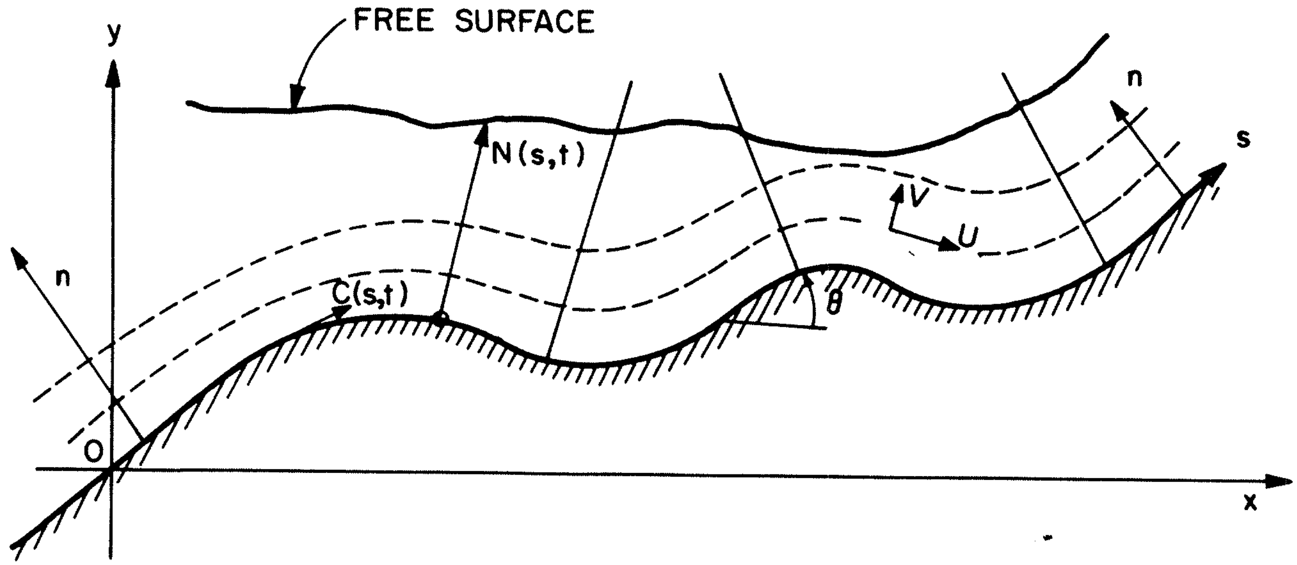


Figure 1. Curvilinear Coordinates Defined by the Channel Boundary ( after Dressler (8) )

Adding the resistance term (8) to Eq. (7), the generalized shallow-flow equations become :

Equation of continuity :

$$\frac{\partial N}{\partial t} + \frac{1}{(1-kN)^2} C \frac{\partial N}{\partial s} - \frac{\ln(1-kN)}{(1-kN)\,k} \frac{\partial C}{\partial s} + \frac{k'}{k^2}\left[\frac{k}{(1-kN)^2} N + \frac{\ln(1-kN)}{(1-kN)}\right] C = 0 \quad , \tag{9}$$

Equation of motion :

$$\frac{\partial C}{\partial t} + \frac{1}{(1-kN)^2} C \frac{\partial C}{\partial s} + g\cos\theta + \frac{k}{(1-kN)^3} C^2 \frac{\partial N}{\partial s} - \left[kg\sin\theta - \frac{k'}{(1-kN)^3} C^2\right] N + g\sin\theta = -\frac{\lambda}{\rho}\frac{C^2}{N}\left[\frac{1}{1-\frac{kN}{2}}\right] \tag{10}$$

These equations show that the velocity is no longer constant over any cross-section orthogonal to a curved bottom. The possible uniform flows are given by $C = \bar{C}$, $N = \bar{N}$ when

$$-\left[kg\sin\theta - \frac{k'}{(1-k\bar{N})^3}\bar{C}^2\right]\bar{N} + g\sin\theta = - - \frac{\bar{C}^2}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right] \tag{11}$$

The sign convention followed is a positive sign for concave curvature ($k > 0$) It is required that this bottom curve be continuous, with continuous slope, and with continuous curvature.

Dressler (8) suggests restricting applications of the equations within the range

$$-0.85 \leqslant kN \leqslant +0.50 \tag{12}$$

Linearization.- The instantaneous tangential bottom velocity and flow depth are now respresented by a steady (mean or base flow) part and a fluctuating component of small amplitude as shown in Fig. 2, and following :

$$C = \bar{C} + C' \qquad N = \bar{N} + N' \tag{13}$$

in which $\bar{C}$ and $\bar{N}$ are the steady parts, and $C'$ and $N'$ are the fluctuating parts of the velocity and flow depth, respectively. By postulate, $C' \ll \bar{C}$ and $N' \ll \bar{N}$.

Substitution of these expressions into Eqs. (9) and (10) results in the differential equations governing the perturbations. The standard linearization assumption of neglecting second order products of the perturbations is made. The justification for this is that provided the amplitudes of the disturbances are sufficiently small, cross terms such as $C'N'$ serve only to generate higher harmonics with greatly reduced amplitudes. However, although the amplitudes of the fluctuations are assumed small, they can be subjected to large amplification rates. The purpose of the linearized perturbation theory is to determine the range of frequencies for which arbitrary disturbances are either attenuated, remain at constant amplitudes, or are amplified.

The following simplifications are now made to Eqs. (9), (10) and (11). Terms such as $1/(1-kN)^q$ are replaced by $1 + qkN$, the first term in a series expansion. The justification for this lies in the typical values that kN achieves in practical applications ($-0.15 \leqslant kN \leqslant +0.15$) such as in channels and spillways. Consequently, terms of order $k^2$ and greater are neglected in this approximation. The $\ell n(\ )$ terms are replaced by the first term in a Taylor series representation. Finally, the effect of variation of k with s is assumed small and is therefore neglected in the analysis.

As a result of the linearization operations, the use of these simplifications and some order of magnitude analyses, the following differential equations governing the perturbations are obtained :

$$\frac{\partial N'}{\partial t} + \bar{C}\,(1+2k\bar{N})\frac{\partial N'}{\partial s} + \frac{\bar{N}}{1-k\bar{N}}\,\frac{\partial C'}{\partial s} = 0 \tag{14}$$

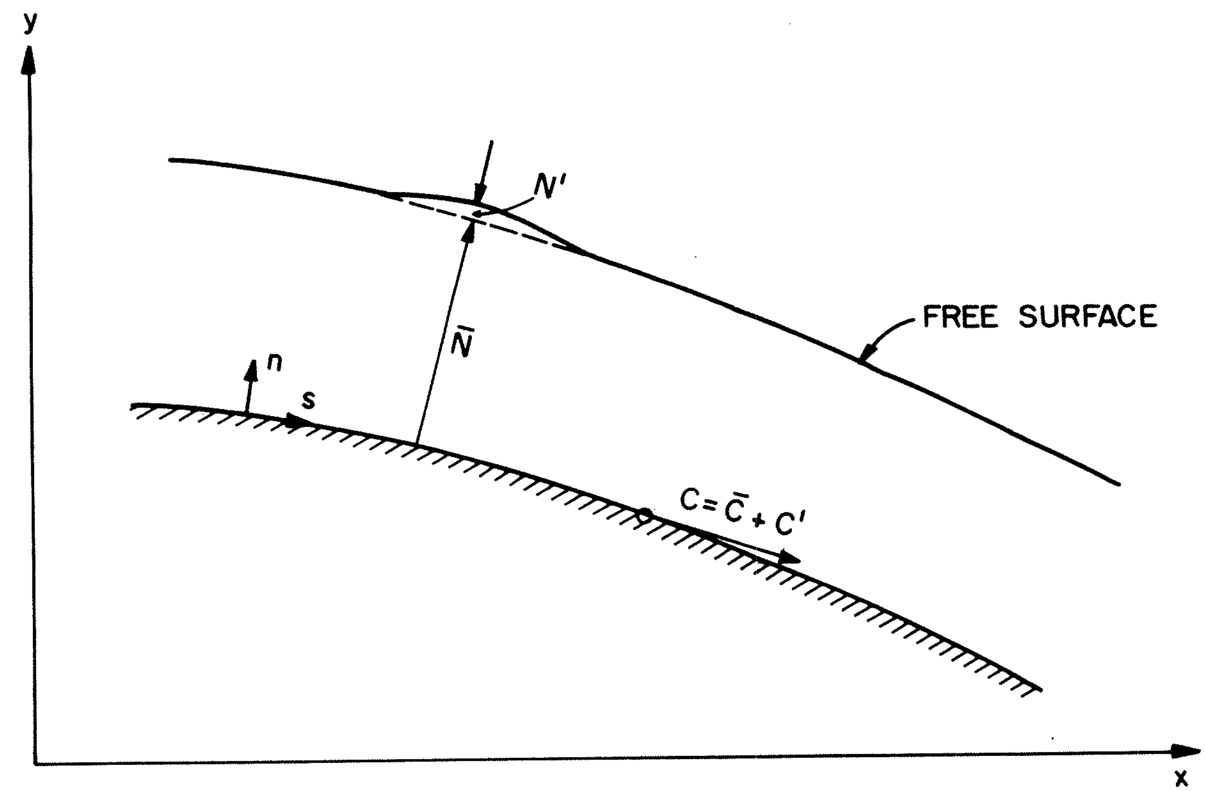


Figure 2. Symbols in Stability Analysis

and

$$\frac{\partial C'}{\partial t} + \bar{C}\,(1+2k\bar{N})\,\frac{\partial N'}{\partial s} + (g\cos\theta + k\,\bar{C}^2)\,\frac{\partial C'}{\partial s}$$

$$= -\,\frac{g\sin\theta\,(1-k\bar{N})}{\bar{N}}\,N' - 2\,\frac{\lambda}{\rho}\,\frac{\bar{C}}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right]C' \qquad (15)$$

The equation describing the possible uniform flows is obtained from Eq. (11). It becomes

$$\bar{C} = \left[-\,\frac{g\sin\theta\;\bar{N}\;(1-k\bar{N})\,(1-\frac{k\bar{N}}{2})}{\lambda}\right]^{1/2}. \qquad (16)$$

Mathematical Form of the Disturbances.- An oscillatory disturbance moving in the s-direction is assumed, and the wave motion may be assumed to consist of Fourier components. A convenient form is :

$$C = C^*(n)\,e^{rt}\,\binom{\cos}{\sin}\left[\beta\left(s + \frac{p}{\beta}\,t\right)\right] \qquad (17)$$

In the above equation, $C^*(n)$ is the amplitude of the superimposed motion, $\beta$ is a wave number, $\frac{p}{\beta}$ is the wave speed and $n$ is the distance measured orthogonally outward from the bottom curve. If $\gamma = r + ip$, then

$$C' = C^*(n)\,\exp\,(\gamma t + i\beta p) \qquad (18)$$

in which $\gamma$ is a complex propagation factor.

Physically, the use of a time-varying perturbation model in the present case implies that at a given location in the flow the disturbances are varying in size with time. Experiments conducted by Liepmann (12) on laminar boundary layers flowing along curved boundaries indicated that instabilities are stationary in time, varying only with streamwise distance. Therefore, a spatially varying disturbance is postulated :

$$C' = C^*(n)\,\exp\left(\int \gamma(s)\,ds + i\beta p\right) \qquad (19)$$

In this formulation, the exponent $\int \gamma(s)\,ds$ is introduced in order to account for any changes in $\gamma$ that occur with $s$. This would be the case, for example, if the curvature $k$ varied in the $s$ direction.

In generalizing the earlier form, an inconsistency is introduced.

If, as in Dressler and Pohle's (6) analysis, the base is a steady uniform flow, flowing parallel to the wall, then $C^*$ is truly a function of n only. However, when, for instance, the curvature (and hence $\Upsilon$) changes, $C^*$ will change. It follows then that this disturbance is not strictly a function of n only. Nevertheless, the assumption of independence of s is believed not to introduce any appreciable error, for the function $C^*(n)$ varies only slightly with $\Upsilon$. Furthermore, Smith (15) ins his work on the growth of Taylor–Goertler vortices along highly concave walls found that terms such as $\partial k/\partial s$ are distinctly minor factors compared with the major terms in the Navier–Stokes equation. This observation is further evidence that neglecting variation with s of $C^*$ has a negligible effect on the solution.

The solution obtained throughout this paper, however, assumes a small rate-of-change of curvature, and hence a constant $\Upsilon$. Therefore, Eq. (18) is used.

Stability Analysis.- It is well known that open-channel flows may become unstable whem the Froude number number is sufficiently high. The subject of whether introduced periodic disturbances may propagate in free-surface flows over curved surfaces, is investigated here using a linear stability analysis similar to that applied by Dressler and Pohle (6). Since Eqs. (14) and (15) form a linear system, they can be reduced to a single equation by eliminating N'. A second order linear equation for C' is obtained.

$$\frac{\partial^2 C'}{\partial t^2} + 2\,\bar{C}\,(1+2k\bar{N})\,\frac{\partial^2 C'}{\partial t\,\partial s} + \left[\bar{C}^2(1+2k\bar{N})^2 - \frac{\bar{N}(g\cos\theta + k\bar{C}^2)}{(1-k\bar{N})}\right]\frac{\partial^2 C'}{\partial s^2}$$
$$+ \left\{2\,\frac{\lambda}{\rho}\,(1+2k\bar{N})\,\frac{\bar{C}^2}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right] - g\sin\theta\right\}\frac{\partial C'}{\partial s}$$
$$+ 2\,\frac{\bar{C}}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right]\frac{\partial C'}{\partial t} = 0 \qquad (20)$$

When Eq. (18) is substituted in Eq. (21),

$$\gamma^2 + \left\{2\bar{C}(1+2k\bar{N})(i\beta) + 2\frac{\lambda}{\rho}\frac{\bar{C}}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right]\right\}\gamma$$

$$+\left\{\left[\bar{C}^2(1+2k\bar{N})^2 - \frac{\bar{N}(g\cos\theta+k\bar{C}^2)}{(1-k\bar{N})}\right](i\beta)^2\right.$$

$$\left.+\left(2\frac{\lambda}{\rho}(1+2k\bar{N})\frac{\bar{C}^2}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right] - g\sin\theta\right)(i\beta)\right\} = 0 \qquad (21)$$

Solution of Eq. (21) implies the following relationship between $\gamma$ and $\beta$

$$\gamma = -\frac{\lambda}{\rho}\frac{\bar{C}}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right] - \bar{C}(1+2k\bar{N})(i\beta) \pm \left\{\frac{\lambda^2}{\rho^2}\frac{\bar{C}^2}{\bar{N}^2}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right]^2\right.$$

$$\left.-\frac{\bar{N}(g\cos\theta+k\bar{C}^2)}{(1-k\bar{N})}\beta^2 + i\beta g\sin\theta\right\}^{1/2} \qquad (22)$$

Stability requires both roots to be in the left half of the complex $\gamma$-p plane. Equating the first square root in Eq. (22) to

$$\frac{\lambda}{\rho}\frac{\bar{C}}{\bar{N}}\left[\frac{1}{1-\frac{k\bar{N}}{2}}\right] + \bar{C}(1+2k\bar{N})(i\beta),$$

separating real and imaginary parts, eliminating $\bar{C}(1+2k\bar{N})\beta$, and using Eq.(16), the condition for the searched root to be on the p-axis is found to be

$$\rho^2(\cos^2\theta - 1)(1-k\bar{N})^4\left(1-\frac{k\bar{N}}{2}\right)^2 = 16\lambda^2\cos^2\theta \qquad (23)$$

Therefore, the instability criterion is

$$\rho^2(\cos^2\theta - 1)(1-k\bar{N})^4\left(1-\frac{k\bar{N}}{2}\right)^2 > 16\lambda^2\cos^2\theta \qquad (24)$$

Using Eq. (16), the instability criterion can be written in general as

$$\bar{C} > 2 \left[ \frac{g \ \bar{N} \cos\theta}{1 - k\bar{N}} \right]^{1/2} \tag{25}$$

when k = 0, expression (25) reduces to the form given by Dressler and Pohle (6), Craya (2) and Vedernikov (17, 18) for a linear channel. If $k < 0$ (convex cha nnel), calling the curvature k, condition (24) becomes

$$\bar{C} > 2 \left[ \frac{g \ \bar{N} \cos\theta}{1 + |k| \ \bar{N}} \right]^{1/2} \tag{26}$$

If $k > 0$ (concave channel), condition (25) holds.

## CONCLUSIONS AND RECOMMENDATIONS

The object of the analysis presented here is to study the hydrodynamic instability of open-channel flow under the simultaneous influence of bottom curvature and Chézy-type resistance. The developed criteria for instability of flow are

for convex channels ( $k < 0$):

$$\bar{C} > 2 \left[ \frac{g \ \bar{N} \ \cos\theta}{1 + |k| \ \bar{N}} \right]^{1/2} \tag{26}$$

and for concave channels ($k < 0$):

$$\bar{C} > 2 \left[ \frac{g \ \bar{N} \cos\theta}{1 - k\bar{N}} \right]^{1/2} \tag{25}$$

These criteria expand yhe finding of Dressler and Pohle (6), Craya (2) and Vedernikov (17,18) by explicitly including the curvature term. A suggested area of further research is to check these criteria by experimental investigations as well as to consider greater values of kN, and cases in whick k' cannot be neglected.

## APPENDIX I - REFERENCES


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## APPENDIX II - NOTATION

The following symbols are used in this paper :

$A$ = cross-sectional area;

$a$ = coefficient;

$b$ = coefficient;

$C$ = tangential velocity along the channel bottom;

$C^*$ = wave amplitude;

$F_c$ = critical Froude number;

$F$ = Froude number;

$f$ = function;

$g$ = gravitational acceleration;

$i$ = $(-1)^{1/2}$;

$j$ = constant;

$m$ = exponent;

$N$ = depth of flow;

$n$ = distance measured orthogonally outwards from the bottom curve;

$P$ = wetted perimeter;

$p$ = parameter;

$Q$ = discharge;

$q$ = exponent;

$R$ = hydraulic radius;

$r$ = parameter;

$S_f$ = friction slope;

$S_o$ = bed slope;

$s$ = length coordinate measured along curved bottom;

$t$ = time;

$U$ = velocity for linear uniform flow;

$u$ = velocity;

$V$ = Vedernikov number;

$v$ = average velocity;

$Y$ = depth for linear uniform flow;

$y$ = flow depth;

$\beta$ = wave number;

$\gamma$ = propagation factor;

$\theta$ = angle of inclination of the channel;

$k$ = curvature;

$k'$ = rate-of-change of curvature;

$\lambda$ = measure of roughness; and

$\rho$ = water density.

Superscripts

$'$ = perturbed variable; and

$\bar{}$ = steady part of the variable.

CURVATURE EFFECTS ON HYDRAULIC INSTABILITY

KEY WORDS : Hydrodynamics, Bores, Roll waves, Hydraulic instability, Curvature, Channels.

ABSTRACT : An investigation on hydrodynamic instability resulting from the combines action of Chézy type of resistance and curvature of the channel floor by using the generalized nonlinear shallow-flow equations developed by Dressler (2) is presented. Linear stability analysis is applied to derive the criteria.

The developed criteria for instability of flow expand the findings of Dressler and Pohle (3), Craya (1) and Vedernikov (4) by including the curvature term.

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