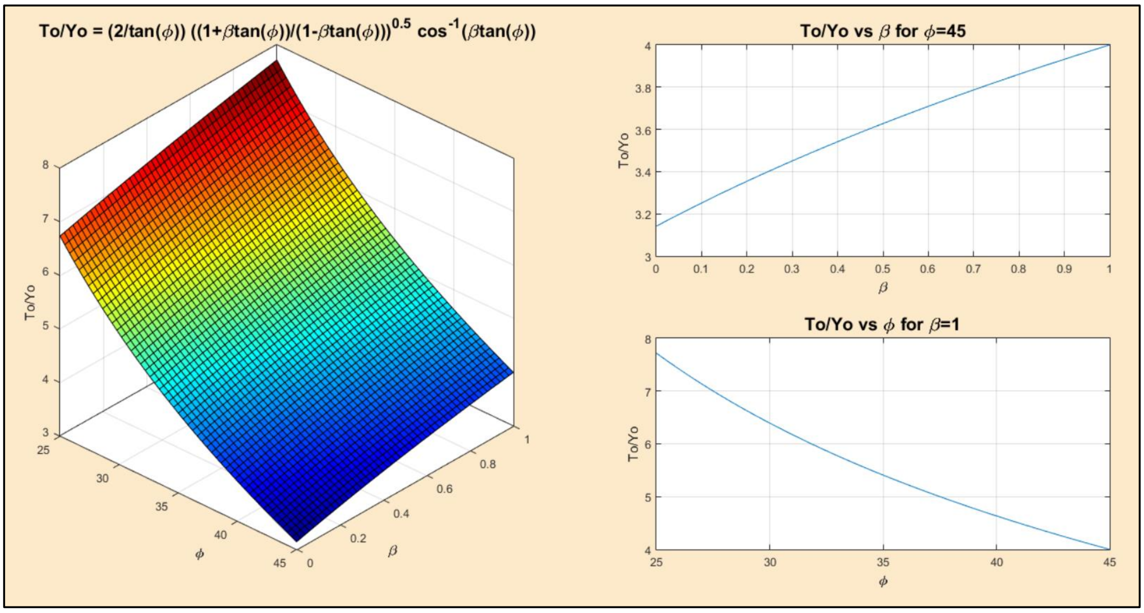
Eq. 57 is:

$$\frac{To}{Yo} = \frac{2}{\tan(\varphi)}\sqrt{\frac{1+\beta\tan(\varphi)}{1-\beta\tan(\varphi)}}cos^{-1}(\beta\tan(\varphi)) \qquad [57]$$



The limit of Eq. 57 for $\beta \to 1$ and $\varphi \to \pi/4$ is:

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = \lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left[\frac{2}{\tan(\varphi)}\sqrt{\frac{1+\beta\tan(\varphi)}{1-\beta\tan(\varphi)}}cos^{-1}(\beta\tan(\varphi))\right]\right]$$

Applying the limit for $\beta \to 1$:

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = \lim_{\varphi\to\pi/4}\left[\frac{2}{\tan(\varphi)}\sqrt{\frac{1+(1)\tan(\varphi)}{1-(1)\tan(\varphi)}}cos^{-1}((1)\tan(\varphi))\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = \lim_{\varphi\to\pi/4}\left[\frac{2}{\tan(\varphi)}\sqrt{\frac{1+\tan(\varphi)}{1-\tan(\varphi)}}cos^{-1}(\tan(\varphi))\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\lim_{\varphi\to\pi/4}\left[\frac{\sqrt{1+\tan(\varphi)}\,cos^{-1}(\tan(\varphi))}{\tan(\varphi)\sqrt{1-\tan(\varphi)}}\right]$$

The expression $\sqrt{1+\tan(\varphi)} = \sqrt{2}$ is taken out of the limit because the indeterminate term "0/0" does not depend on $\sqrt{1+\tan(\varphi)}$. Therefore:

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{cos^{-1}(\tan(\varphi))}{\tan(\varphi)\sqrt{1-\tan(\varphi)}}\right]$$

Applying L'Hospital's rule:

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\frac{\frac{d}{d\varphi}\left[cos^{-1}(\tan(\varphi))\right]}{\frac{d}{d\varphi}\left[\tan(\varphi)\sqrt{1-\tan(\varphi)}\right]}$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{sec^2(\varphi)}{\sqrt{1-\tan(\varphi)^2}}}{sec^2(\varphi)\sqrt{1-\tan(\varphi)}-\frac{\tan(\varphi)sec^2(\varphi)}{2\sqrt{1-\tan(\varphi)}}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{1}{\sqrt{1-\tan(\varphi)^2}}}{\sqrt{1-\tan(\varphi)}-\frac{\tan(\varphi)}{2\sqrt{1-\tan(\varphi)}}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{1}{\sqrt{1-\tan(\varphi)^2}}}{\frac{2(1-\tan(\varphi))-\tan(\varphi)}{2\sqrt{1-\tan(\varphi)}}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{1}{\sqrt{1-\tan(\varphi)^2}}}{\frac{2-3\tan(\varphi)}{2\sqrt{1-\tan(\varphi)}}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{1}{\sqrt{(1+\tan(\varphi))}\sqrt{(1-\tan(\varphi))}}}{\frac{2-3\tan(\varphi)}{2\sqrt{1-\tan(\varphi)}}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\lim_{\varphi\to\pi/4}\left[\frac{-\frac{1}{\sqrt{1+\tan(\varphi)}}}{\frac{2-3\tan(\varphi)}{2}}\right]$$

Substituting $\varphi = \pi/4$:

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\left[\frac{-\frac{1}{\sqrt{1+\tan(\pi/4)}}}{\frac{2-3\tan(\pi/4)}{2}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\left[\frac{-\frac{1}{\sqrt{1+1}}}{\frac{2-3}{2}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 2\sqrt{2}\left[\frac{2}{\sqrt{2}}\right]$$

$$\lim_{\beta\to 1}\left[\lim_{\varphi\to\pi/4}\left(\frac{To}{Yo}\right)\right] = 4 \qquad [60]$$

Therefore, it is proven that *To/Yo* = 4 when β =1 and $\varphi = \pi/4$.