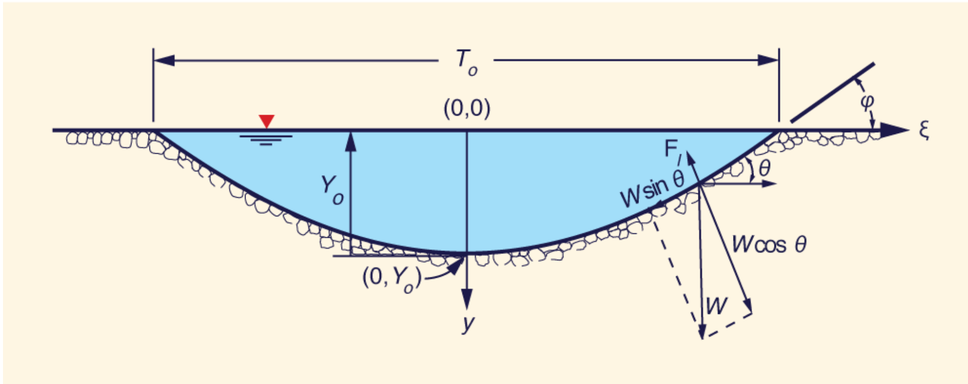

Definition sketch for the shape of a self-formed channel in noncohesive alluvium.

WHAT IS THE ASPECT RATIO OF A SELF-FORMED ALLUVIAL STREAM?

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ABSTRACT. The aspect ratio of a self-formed stream in noncohesive alluvium has been calculated using the Lane and others (1959) theory. The aspect ratio T_o / Y_o is expressed in terms of the friction angle ϕ of the surrounding soil and the lift-to-drag force ratio β acting on each particle. The results agree very well with practical experience. An online calculator may be used to show the sensitivity of the calculated results to the input variables ϕ and β.

1. INTRODUCTION

Alluvial streams are self-formed, i.e., their shape, characterized by depth and width, obeys the laws of mechanics, properly within the field of river mechanics. The ratio of width to depth is referred to as the *aspect ratio* of the cross-section. The study of alluvial river flow encompasses the calculation of three variables: (1) depth, (2) width, and (3) slope. Depth and slope may be calculated using the laws of fluid mechanics, reflected in the fundamental laws of conservation of mass, momentum, and energy (Chow, 1959; **Ponce, 2014**).

Momentum integrates the acting force in time, while energy integrates the same force in space; therefore, they are not independent. In steady flow in open channels, the equations of mass and energy are used to solve for the unknown flow variables (depth and slope). In unsteady flow, the equations of mass and momentum are used for the same purpose. This reality leaves the calculation of stream width out of fluid mechanics. The width would have to be calculated using principles of river mechanics.

Lane and others (1959) developed a mechanistic approach to calculate the width in a self-formed channel. The theory has been recently revisited by **Ponce and Jiang (2020)**. While Lane and others' approach is limited to noncohesive materials under equilibrium conditions, it is a point-of-start in the nascent field of unsteady alluvial river mechanics.

The application of Lane's methodology may serve to throw additional light onto relevant issues of floods, flood damages and related sedimentation issues (ASCE, 1975; 2007). Moreover, in light of contemporary global climate change, the subject is now becoming increasingly important.

2. LANE AND OTHERS' THEORY

The size of the cross-section for a stream channel in equilibrium, formed with noncohesive materials, is dependent on the maximum threshold discharge. Using principles of river mechanics, Lane and others (1959) showed that the shape of the equilibrium cross section is a function of the following variables: (1) the friction angle ϕ of the material forming the channel bed, and (2) the lift-to-drag force ratio β acting on a particle.

The main assumptions used by Lane and others in deriving their theory are the following:

1. The side slope of the channel, at or above the water surface, is equal to the friction angle ϕ of the noncohesive alluvial material (Fig. 1).

2. The particles at the channel boundary are at the condition of incipient motion. The lift and drag forces acting on each particle, together with the downward component of the gravity force, are balanced exactly by the frictional force developed between particles. The lift and drag forces are assumed to be proportional to the tractive force exerted on the channel bottom. The *lift-to-drag force ratio* is referred to as β.

3. The particles are held in place by virtue of their submerged weight resolved in a direction normal to the bed.

4. The tractive force acts in the direction of the flow and is equal to the weight of the column of water above the area on which the force acts.

5. The channel is assumed to be prismatic, with negligible secondary currents and the absence of backwater effects.

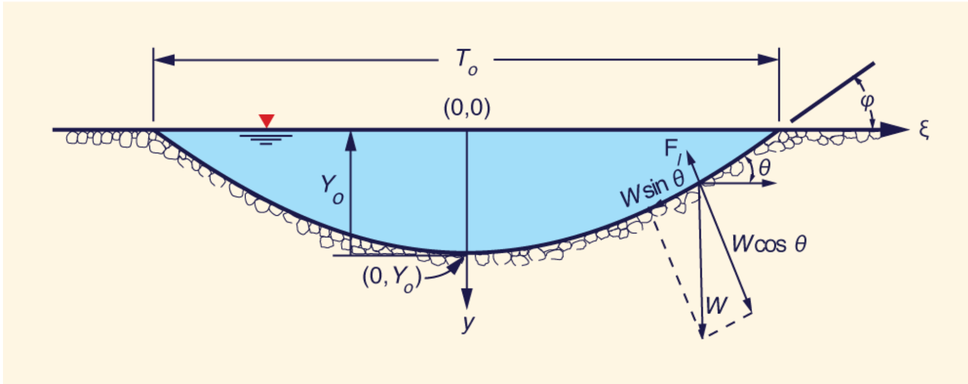

Fig. 1 Definition sketch for the shape of a self-formed channel in noncohesive alluvium.

3. FINDINGS

The derivation of the Lane and others (1959) theory has been extensively documented by **Ponce and Jiang (2020): Section 3**. In that report, Eq. 57, reproduced here as Eq. 1, is the equation derived by Lane and others for the width-to-depth ratio, or aspect ratio of a self-formed channel carved in noncohesive material under equilibrium conditions:

$$\frac{T_o}{Y_o} = \frac{2}{\tan\phi} \left(\frac{1+r}{1-r}\right)^{1/2} \cos^{-1} r \qquad (1)$$

in which T_o = channel top width, Y_o = maximum (center) flow depth, ϕ = friction angle of the noncohesive material, β = lift-to-drag force ratio, and $r = \beta \tan\phi$.

Table 1 shows aspect ratios calculated using Eq, 1, for friction angle varying in the range $5 \le \phi \le 45°$, at intervals of 5°, and lift-to-drag-force ratio varying in the range $-1 \le \beta \le 1$, at intervals of 0.1. For a typical value of $\phi = 30°$ and $\beta = 0.4$ (Apperley, 1968), the aspect ratio is: $T_o / Y_o = 5.86$. Likewise, for $\phi = 45°$ and $\beta = 1$: $T_o / Y_o = 4$.

Table 1. Value of T_o / Y_o as a function of friction angle ϕ and lift-to-drag force ratio β.

β	Friction angle ϕ (degrees)								
	5	10	15	20	25	30	35	40	45
-1.0	34.728	16.591	10.447	7.292	5.320	3.920	2.814	1.809	0.001
-0.9	34.849	16.722	10.589	7.448	5.495	4.124	3.064	2.160	1.235
-0.8	34.970	16.851	10.728	7.598	5.660	4.310	3.284	2.438	1.665
-0.7	35.090	16.978	10.862	7.743	5.817	4.484	3.480	2.672	1.971
-0.6	35.210	17.103	10.994	7.882	5.966	4.645	3.658	2.874	2.214
-0.5	35.328	17.226	11.123	8.017	6.109	4.796	3.822	3.054	2.418
-0.4	35.446	17.348	11.248	8.148	6.245	4.939	3.973	3.216	2.595

-0.3	35.563	17.467	11.371	8.274	6.375	5.074	4.113	3.364	2.752
-0.2	35.679	17.585	11.492	8.397	6.500	5.202	4.245	3.500	2.894
-0.1	35.794	17.702	11.609	8.516	6.621	5.325	4.369	3.626	3.023
0.0	35.909	17.817	11.725	8.631	6.737	5.441	4.487	3.744	3.142
0.1	36.022	17.930	11.838	8.744	6.849	5.553	4.598	3.855	3.252
0.2	36.135	18.042	11.948	8.854	6.958	5.660	4.704	3.959	3.354
0.3	36.248	18.153	12.057	8.961	7.063	5.763	4.805	4.058	3.451
0.4	36.359	18.262	12.164	9.065	7.165	5.863	4.902	4.152	3.542
0.5	36.470	18.370	12.269	9.167	7.264	5.959	4.994	4.241	3.628
0.6	36.580	18.476	12.371	9.266	7.360	6.051	5.083	4.327	3.709
0.7	36.690	18.581	12.473	9.363	7.453	6.141	5.169	4.408	3.787
0.8	36.799	18.685	12.572	9.458	7.544	6.227	5.252	4.487	3.861
0.9	36.907	18.788	12.670	9.551	7.632	6.311	5.331	4.562	3.932
1.0	37.014	18.889	12.766	9.642	7.718	6.393	5.408	4.635	4.000

4. ONLINE CALCULATION

Equation 1 may be solved with the aid of the online calculator **ONLINE LANE SHAPE**. For instance, for Y_o = 1 m, ϕ = 30°, and β = 0.4, the answer is: T_o = 5.8627 m, which is in agreement with the result shown in Table 1.

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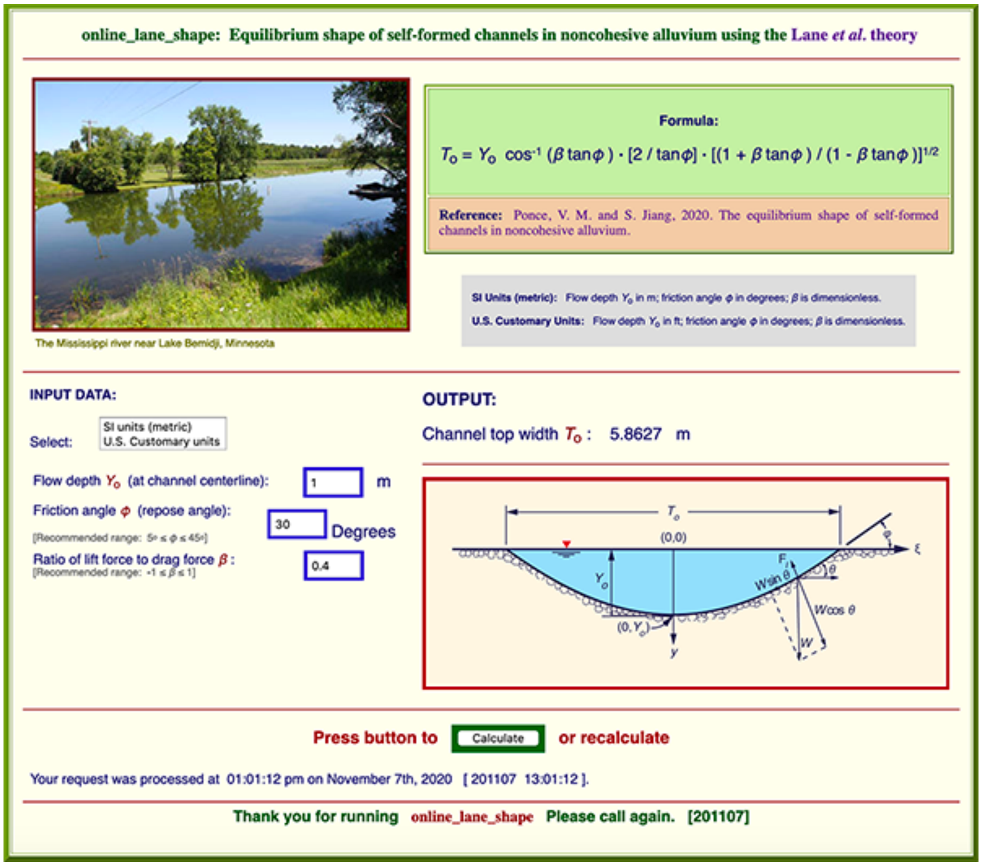

5. CONCLUDING REMARKS

The aspect ratio of a self-formed stream in noncohesive alluvium has been calculated using the Lane and others (1959) theory. The aspect ratio T_o / Y_o is expressed in terms of the friction angle ϕ of the material forming the channel bed and the lift-to-drag force ratio β acting on each particle. The results are seen to agree very well with practical experience. An online calculator may be used to show the sensitivity of the calculated results to the input variables ϕ and β.

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